INTRODUCTION TO PDE'S AND SPECTRAL THEORY

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The aim of spectral theory is to "diagonalise" some operators in infinite-dimensional spaces, by finding eigenfunctions and eigenvectors. The typical example of an operator which we want to diagonalise is the Laplacian, in a compact or a non-compact domain. This course will be an introduction to spectral theory, and its applications to linear partial differential equations.

In the first chapter, we will recall the facts we will need about Sobolev spaces and elliptic equations, not giving all the proofs. Good references on the subject include [1, §5,§6], [2] or [3, §9].

We will then prove the spectral theorem for compact and non-compact self-adjoint operators. If time allows, we will also explain how scattering theory allows one to have more spectral information on Schrödinger operators in $\mathbb{R}^d$.

a) Sobolev spaces and elliptic equations.
b) Spectral theory of compact self-adjoint operators. Introduction to spectral geometry.
c) Spectral theory of non-compact self-adjoint operators.
d) Introduction to scattering theory.

References

[1] L.C. Evans, *Partial differential equations*, Graduate studies in mathematics, **19** American mathematical society **264** (1998).
[2] P. Grisvard, *Elliptic problems in nonsmooth domains*, volume 24 of Monographs and Studies in Mathematics. Pitman. (1985).
[3] H. Brezis, (2010). *Functional analysis, Sobolev spaces and partial differential equations* (2010).