Hyperbolic PDEs and Mechanics

S. Junca, L. Monasse

This lecture is a mathematical introduction to hyperbolic PDEs. The classical mathematical theory is presented first for the scalar case,

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0.$$

In general, the solution becomes not smooth for nonlinear fluxes f. Shock waves appear in finite time. Thus, weak solutions are considered. The physical notion of entropy solution selects an unique solution in a mathematical framework. Then hyperbolic systems are studied. Important examples from fluid and solid mechanics are studied, explained and visualized,

- transport equation $\partial_t u + c\,\partial_x u = 0$,
- Burgers equation $\partial_t u + \partial_x u^2/2 = 0$,
- vibrating strings $\partial_t^2 u - c^2\,\partial_x^2 u = 0$,
- nonlinear elasticity $\partial_t \varepsilon = \partial_x v,\ \partial_t v = \partial_x \sigma(\varepsilon)$.

Open problems and recent mathematical tools are also presented. In particular, some challenges for multidimensional problems are exposed.

The following self contained book is suggested for the one-dimensional case,

[B] Bressan, Alberto, **Hyperbolic systems of conservation laws. The one-dimensional Cauchy problem**. Oxford Lecture Series in Mathematics and its Applications, 20. Oxford University Press, Oxford, 2000. xii+250 pp.

Other references are given during the course.

Mathematical background: analysis, differential calculus, ordinary differential equations.