Ph.D. research topic

- Title of the proposed topic (en anglais) : COMPUTATIONAL GEOMETRY LEARNING IN NON-LINEAR SPACES
- Post-doc
- Research axis of the 3IA: Core elements of AI
- Supervisor (name, affiliation, email) Jean-Daniel Boissonnat, Inria, jean-daniel.boissonnat@inria.fr
- Potential co-supervisor (name, affiliation): Clément Maria, Inria
- The laboratory and/or research group : DataShape project-team, Inria Sophia Antipolis - Méditerranée

- The description of the topic

Computational geometry learning is the field of research that consists of inferring geometric and topological properties of shapes known only partially. A main case of study is the inference of properties of manifolds, embedded in high-dimensional Euclidean space, known only by point samples.

In the past 20 years, deep mathematical and algorithmic techniques have been introduced to address this problem, such as the theory of persistent homology to infer the homology of shapes presented by noisy samples, applications of optimal transport and statistics to the de-noising of geometric data, and exact homeomorphic reconstruction of sub-manifolds presented by samples, to name a few.

A main field of application of these fundamental techniques is data analysis. Intuitively, understanding the shape of data gives insights on the number of its connected components—similarly to clustering—but also higher orders of connectivity, such as filaments, holes, and voids in the way data spreads in space.

More concretely, geometric and topological descriptors of data, obtained by geometry learning techniques, have proved useful and complementary to other descriptors in solving general learning problems, in various fields of applied mathematics such as material science, shape classification, or clustering.

All approaches mentioned above focus on the learning of shapes embedded in Euclidean space. However, the Euclidean metric is restrictive, and in particular does not capture the full complexity of real life data that complex non-linear systems generate. For example, data embedded in general manifolds and described by non-Euclidean metrics are found in molecular chemistry, complex solid modelling, quantum chaos theory and cosmic topology, or neuro-mathematics. In general, the embedding in manifolds with non-trivial topology and
the non-Euclidean metric are both inherited from the intrinsic symmetries and the physical nature of the dynamical system the data describe.

Consequently, the goal of this project is to extend the methods of computational geometry learning to the inference of geometric and topological properties of shapes embedded in general non-linear spaces.

The challenges are twofold. On the one hand, the candidate will have to extend the computational learning techniques to non-Euclidean geometries (axis I). On the other hand, shapes embedded in general spaces are constrained by the topology of the embedding space, which must be taken into account when solving geometric problems in the embedding space. As a consequence, the topology of the embedding space must be computed and described to allow computation in the space (axis II).

For more detail see https://www-sop.inria.fr/members/Jean-Daniel.Boissonnat/resources/jdb/postdoc3IA.pdf