Towards useful quantum computation

Damian Markham

Plan

1. What is quantum?

2. What is quantum computing?

3. What can we do with quantum computers?

4. What's so hard about building a quantum computer?

5. Near term quantum computing

6. Going beyond big algorithms: quantum networks

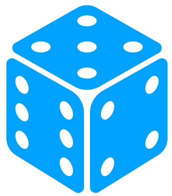
1. What is quantum?

1. What is quantum?

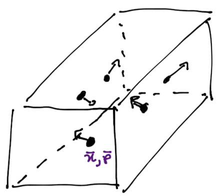
Quantum randomness is different from classical 'randomness'

Classical randomness = ignorance!

E.g. - rolling a die



- Boltzmann distribution of particles in a box



If we *know* the initial conditions, outcome is **deterministic**

Quantum randomness $\neq$ ignorance!

Quantum randomness ≠ ignorance!

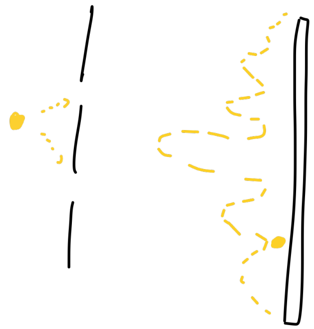
Polarisation filter measurements

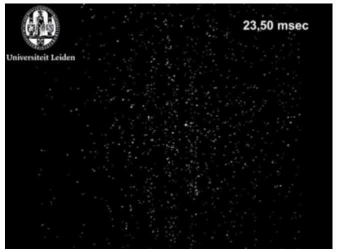
Sunglasses, photographic plates….

The tilting head game: (try looking at your phone / tablette
through polarised sunglasses and tilt you head)

Polarisation filter measurements

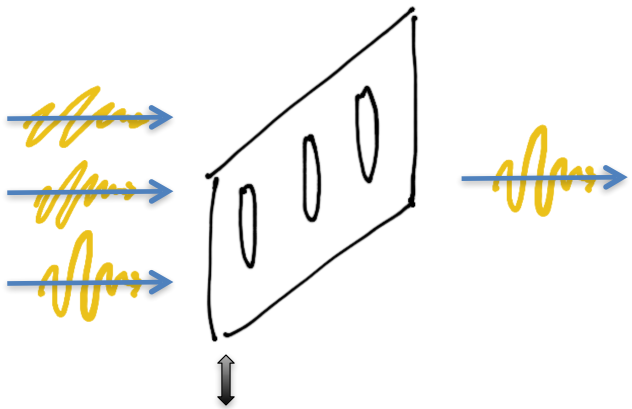
- Light comes in single photons



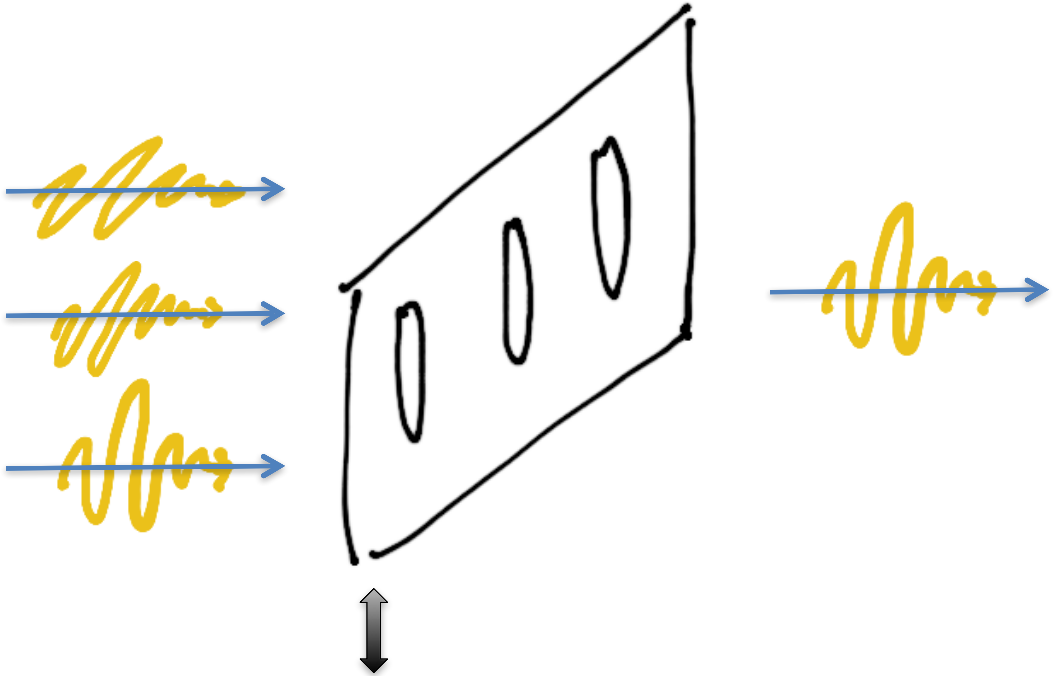


https://physicsworld.com/a/frog-photoreceptor-counts-photons/

- Polarizing filters: only aligned photons pass

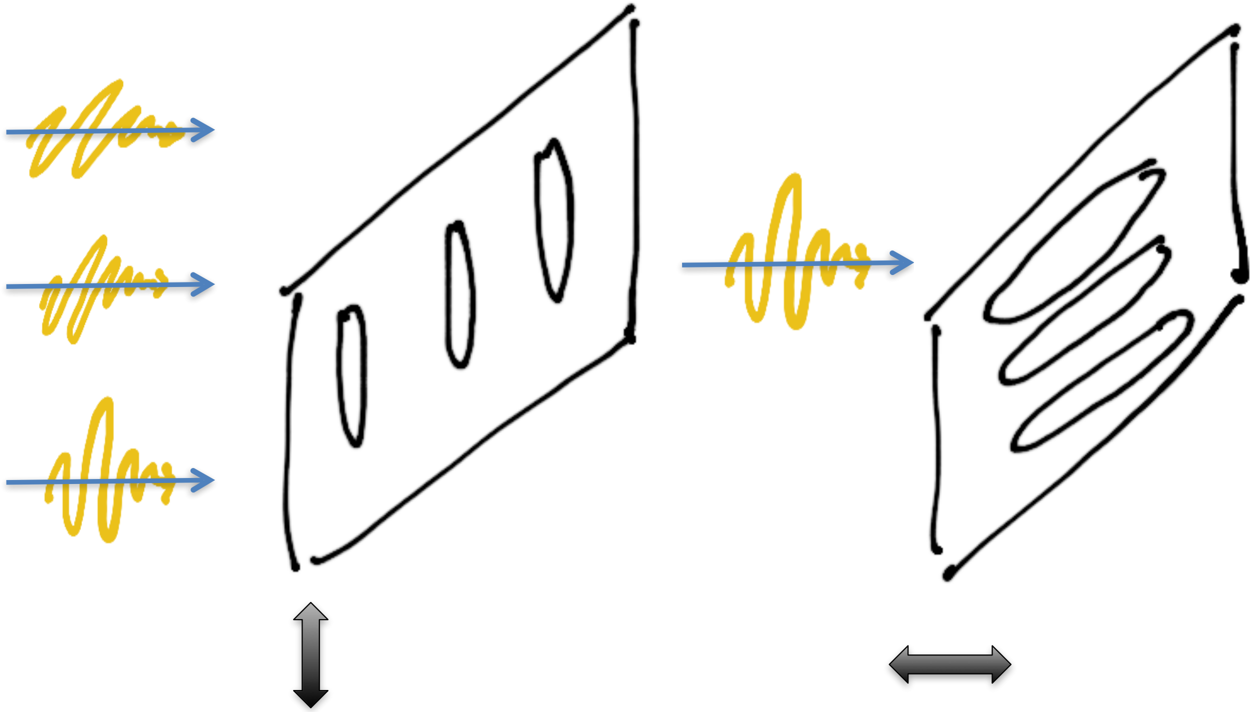


Polarisation filter measurements



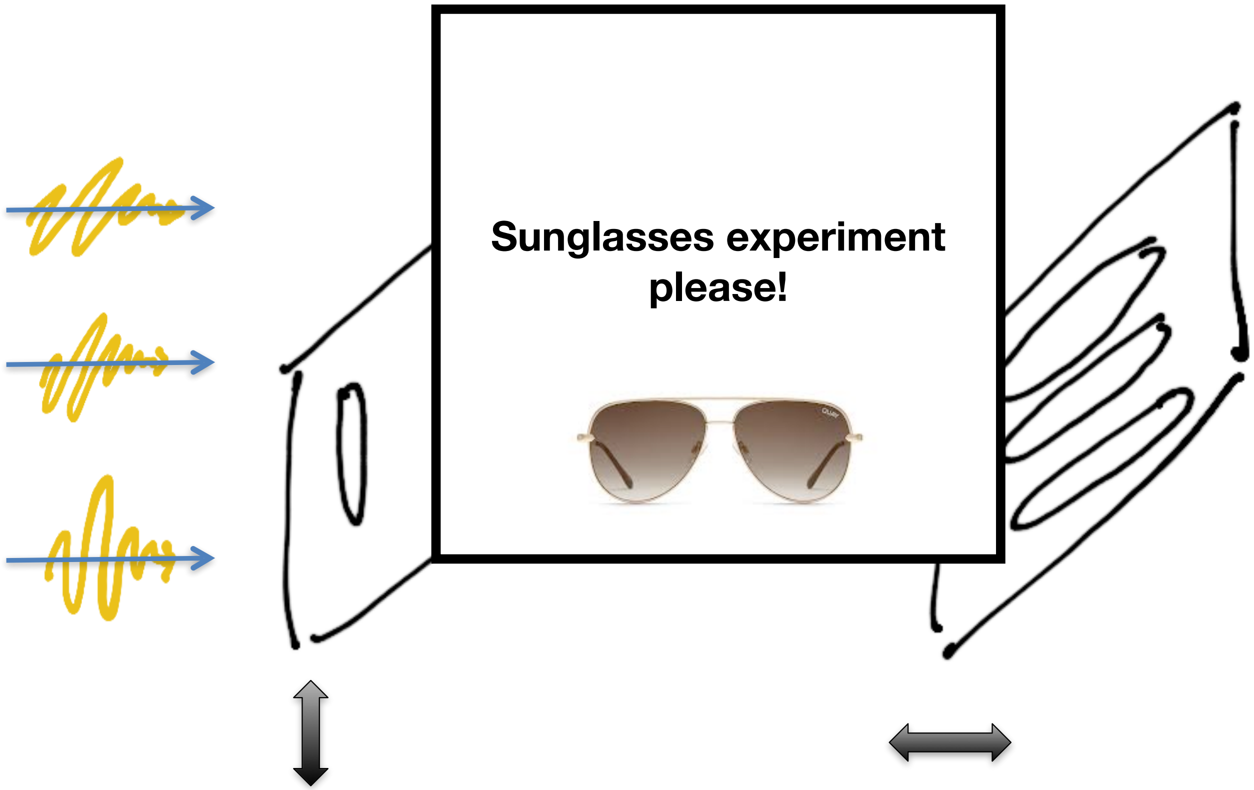
Only light polarised in fixed direction passes through

Combining filters -> less light



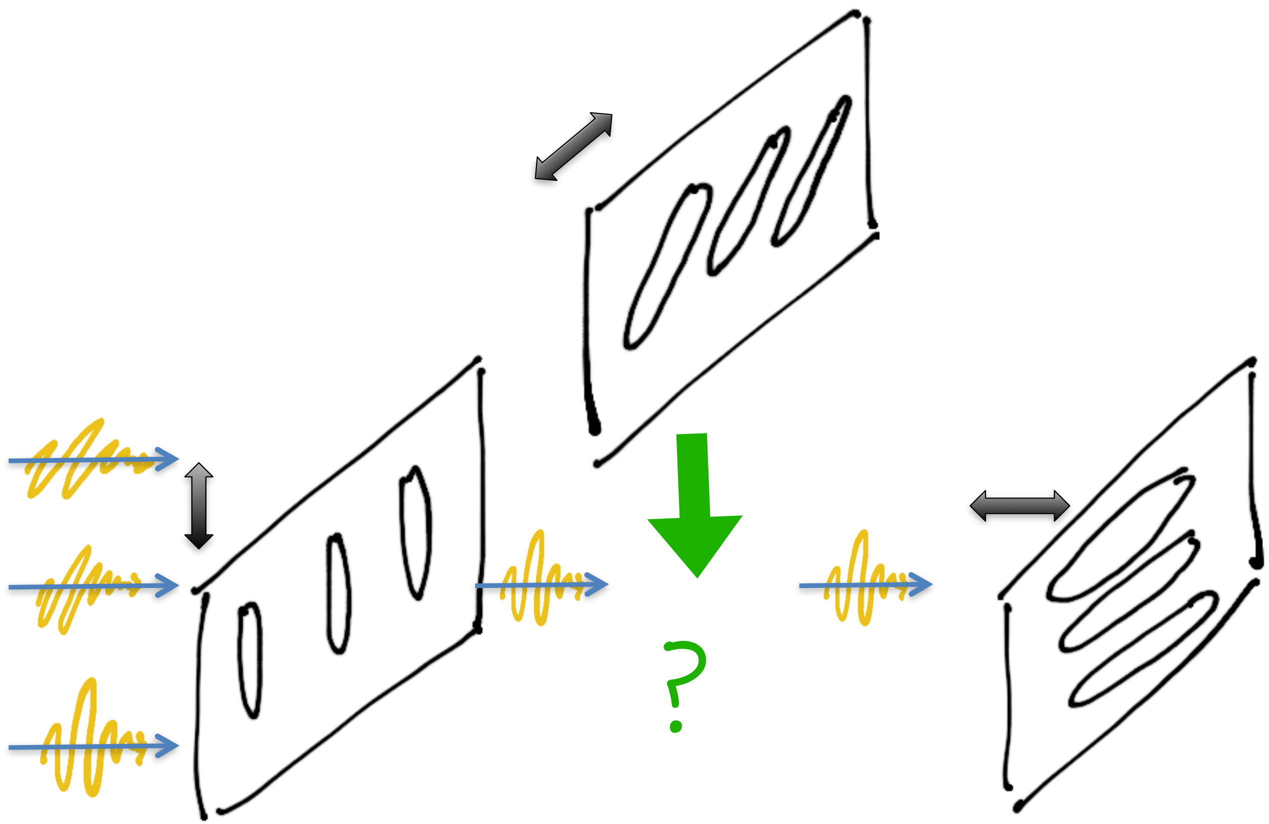
Sequence of perpendicular filters -> NO LIGHT gets ghrough

Combining filters -> less light



Sequence of perpendicular filters -> NO LIGHT gets ghrough

Inserting a filter…?



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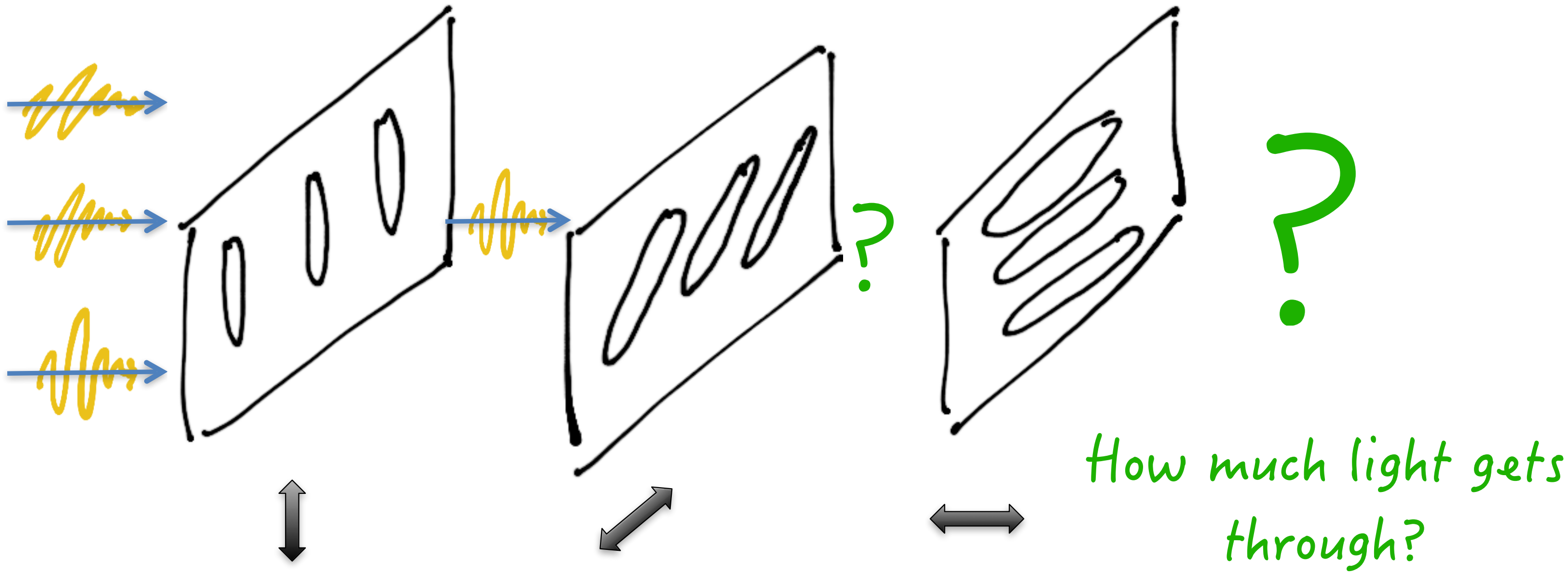

Polarisation filter measurements:

A classical model

- Light comes in single photons

Classical assumptions:

- The ‘measurement’ is deterministic (modulo our ignorance)

- Measurements do not change the system

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Individual photon should either go through or get absorbed determinsitically!

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Having an extra filter in between should not effect this property

Polarisation filter measurements:

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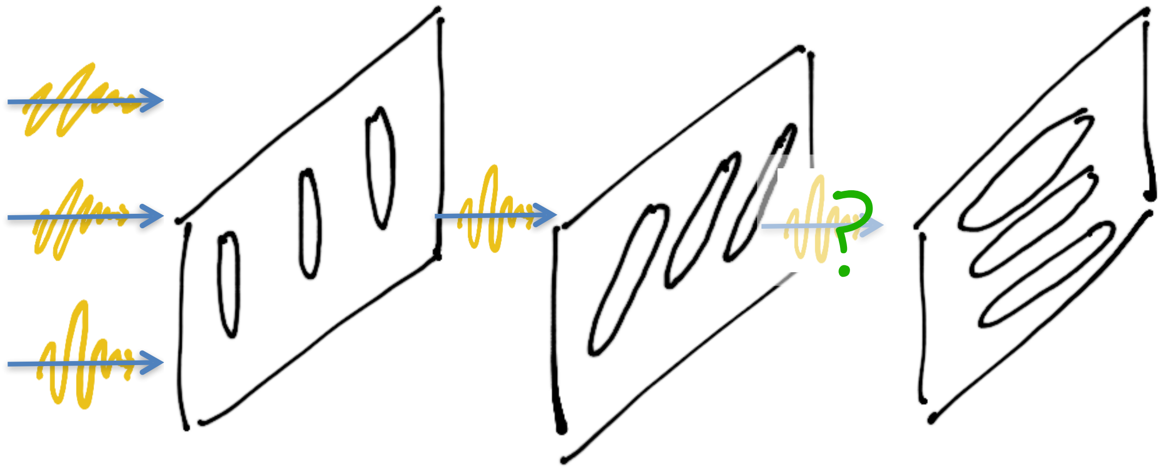

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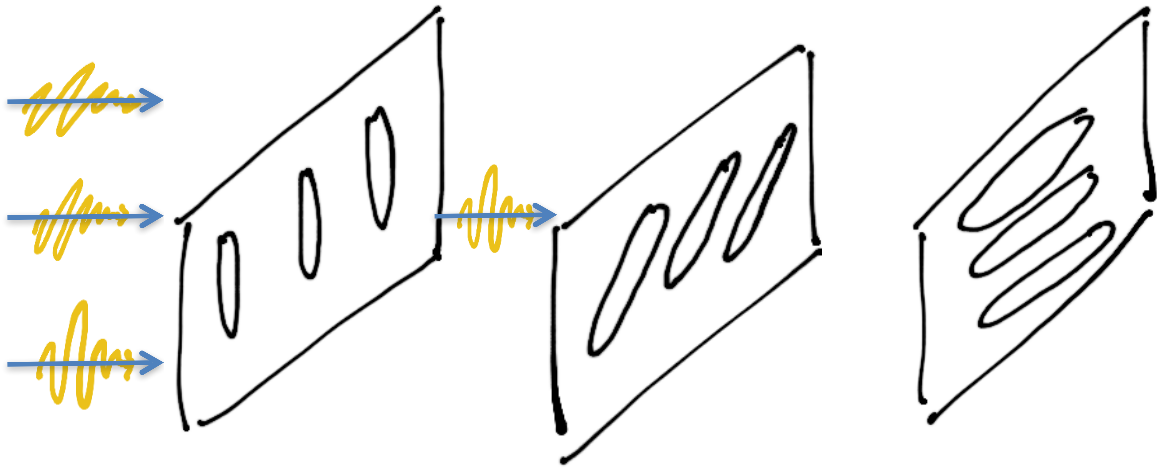
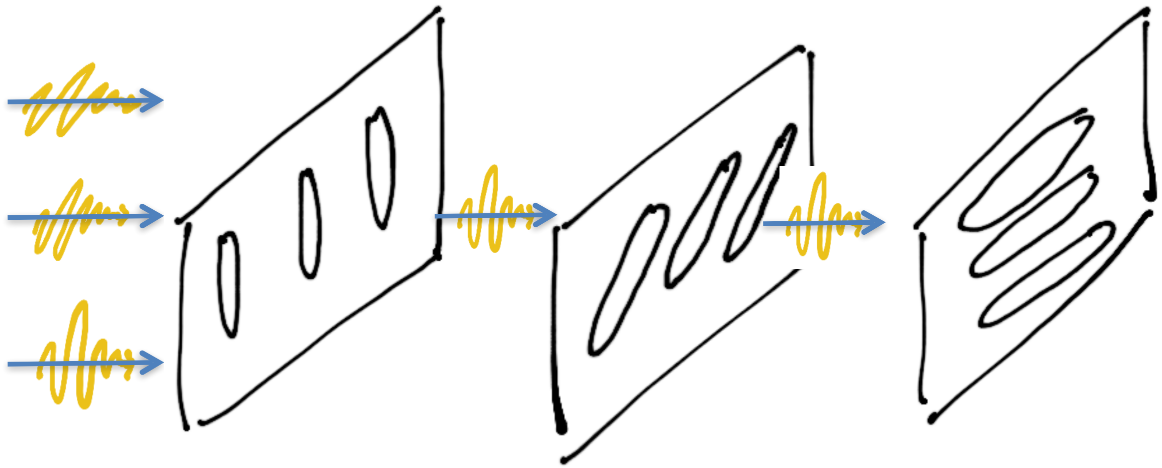


-> if absorbed, <u>no photon out</u>

Polarisation filter measurements:

A classical model

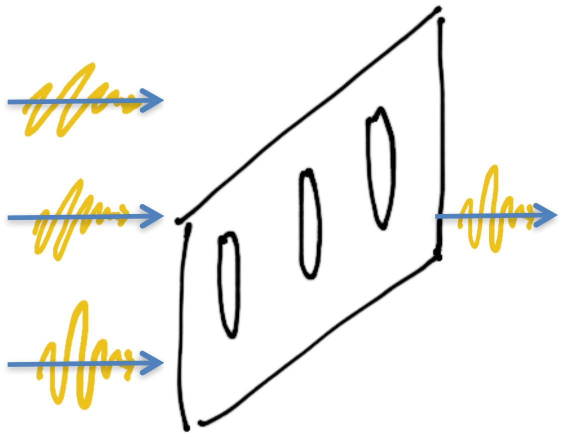


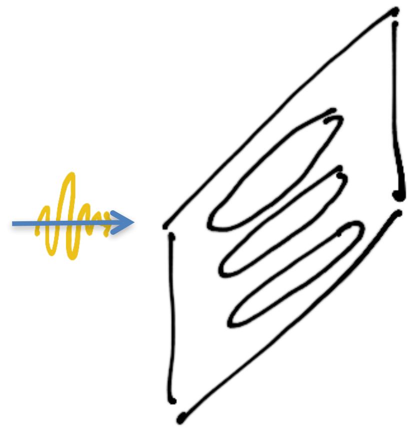
-> if not absorbed

Having an extra filter in between should not effect this property

Polarisation filter measurements:

A classical model





-> if not absorbed

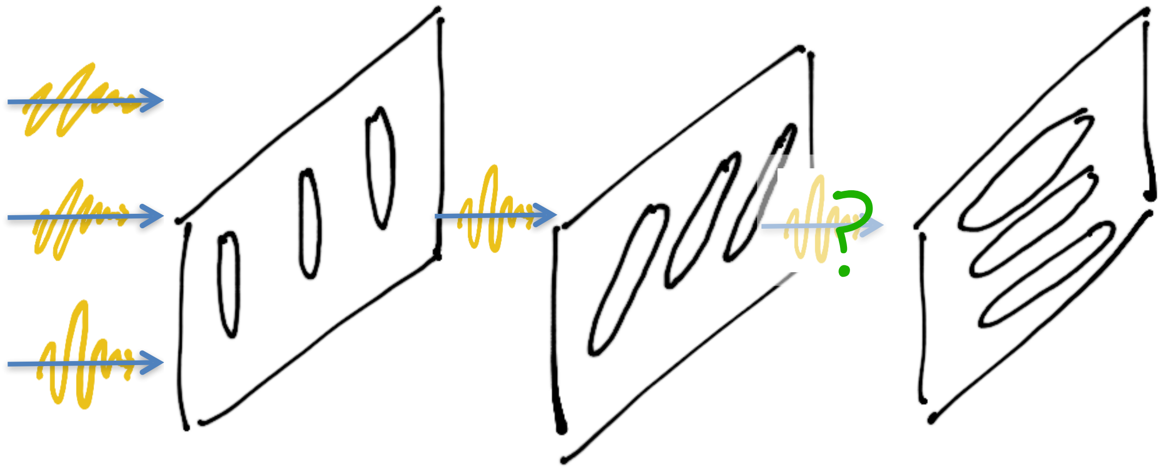
Having an extra filter in between should not effect this property

...as if no filter...

-> <u>no photon out</u>

Polarisation filter measurements:

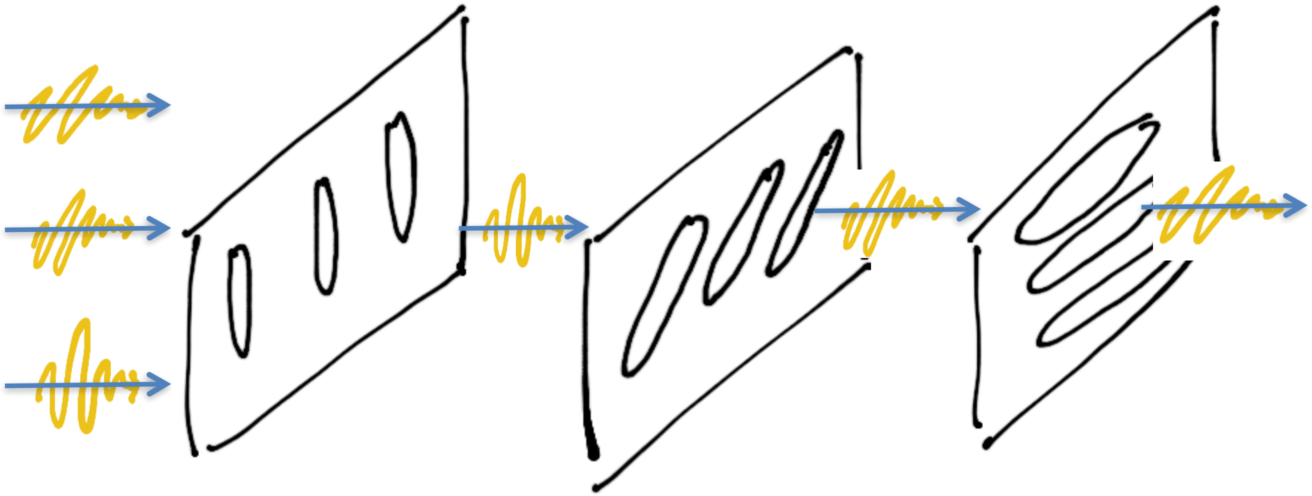
A classical model



Classically: in all cases
-> no photon out

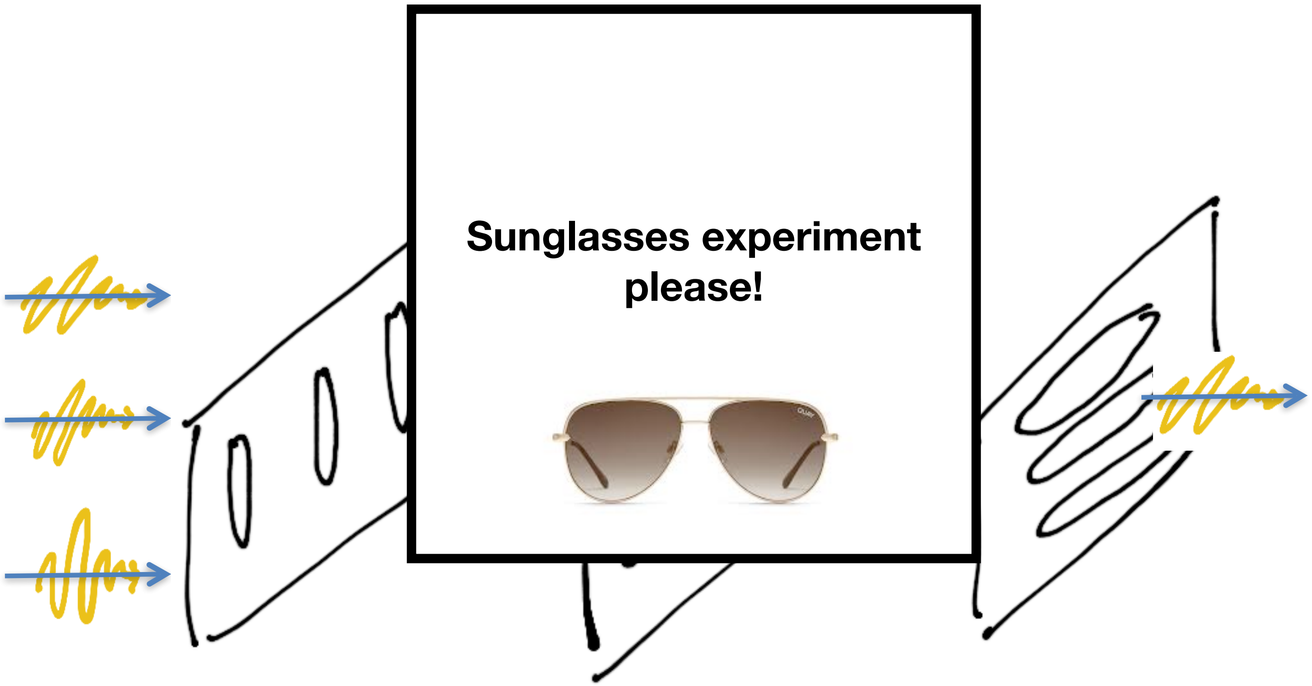
In the real ‘quantum world…

In the real ‘quantum world…



Adding a filter
-> some photons out!

In the real 'quantum world...



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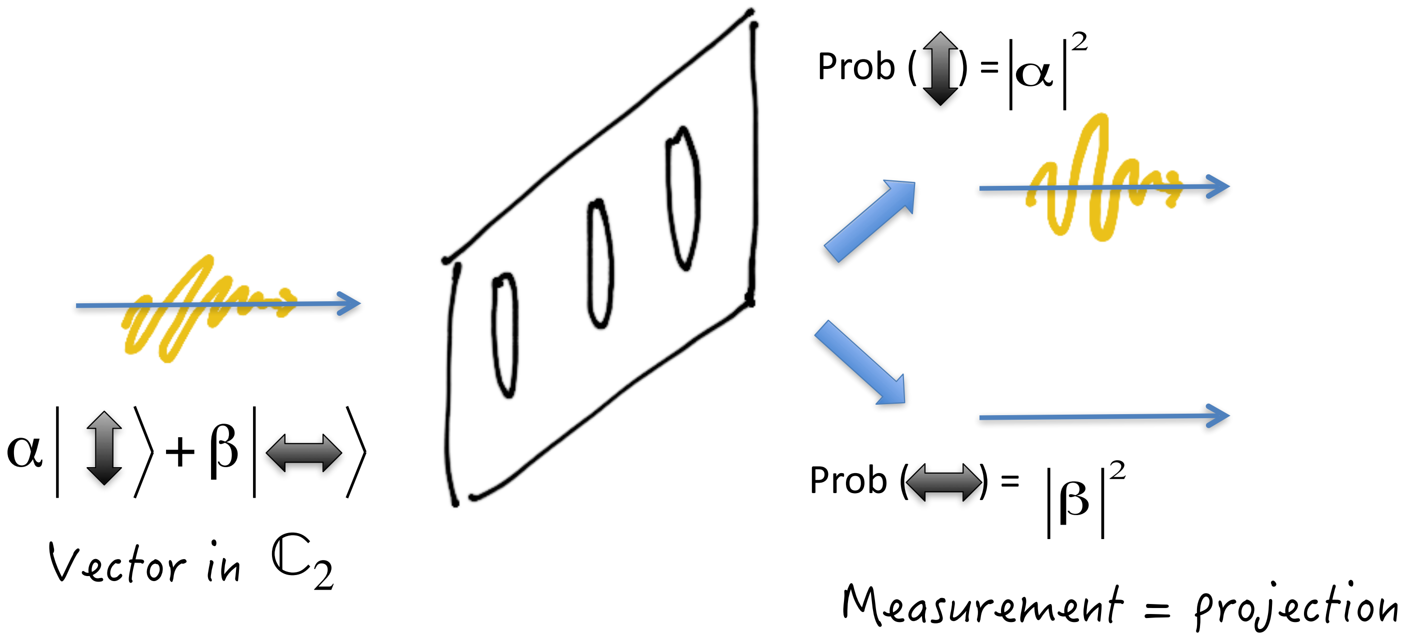
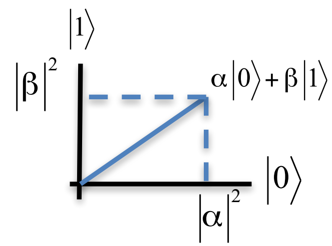
Polarisation filter measurements:

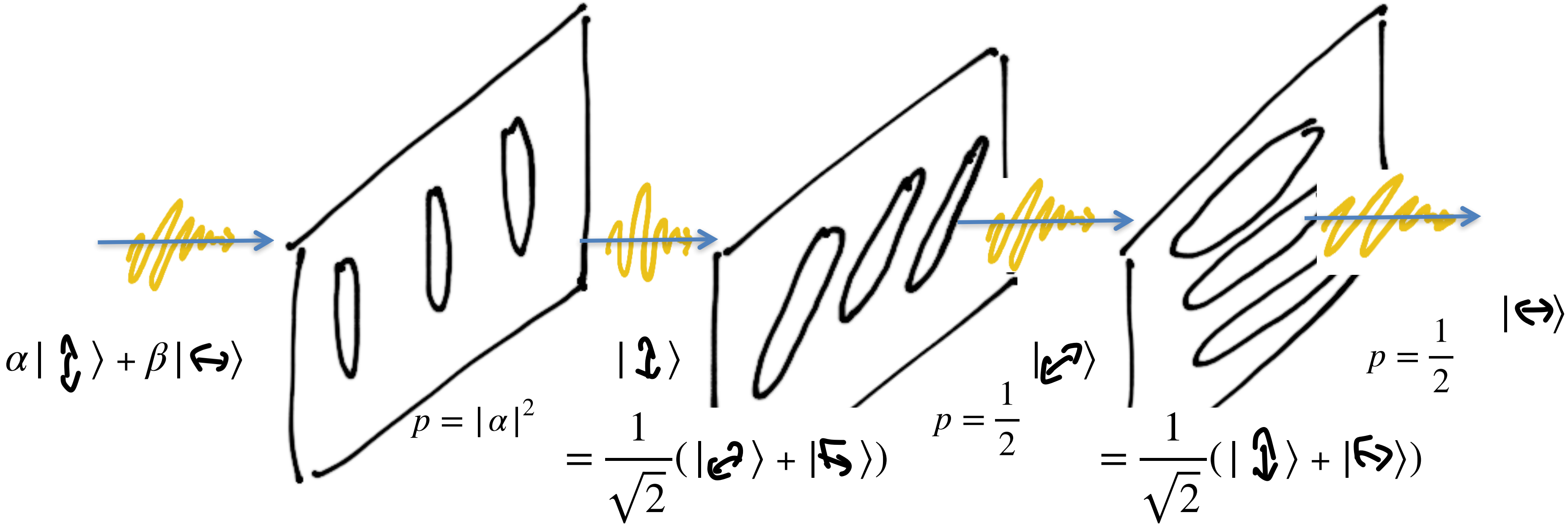
Quantum measurements



- Out come is random
- Output state depends on measurement result



QM answer: Filters as a measurement



Some easy linear algebra…

Adding a filter
-> <u>some photons out!</u>

Quantum randomness is not just ignorance!

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There is no way to assign ‘value’ to the polarisation and get a deterministic outcome

Polarisation filter measurements

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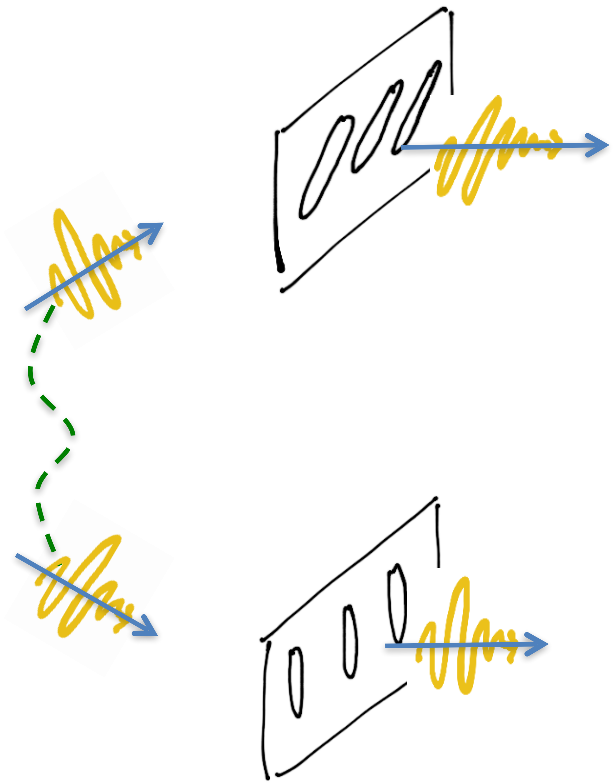
Individual photon should either go through or get absorbed **determinsitically!**

- Measurements do not change the system

~~Having an extra filter in between should not effect this property~~

I don't believe it!

Bell's theorem

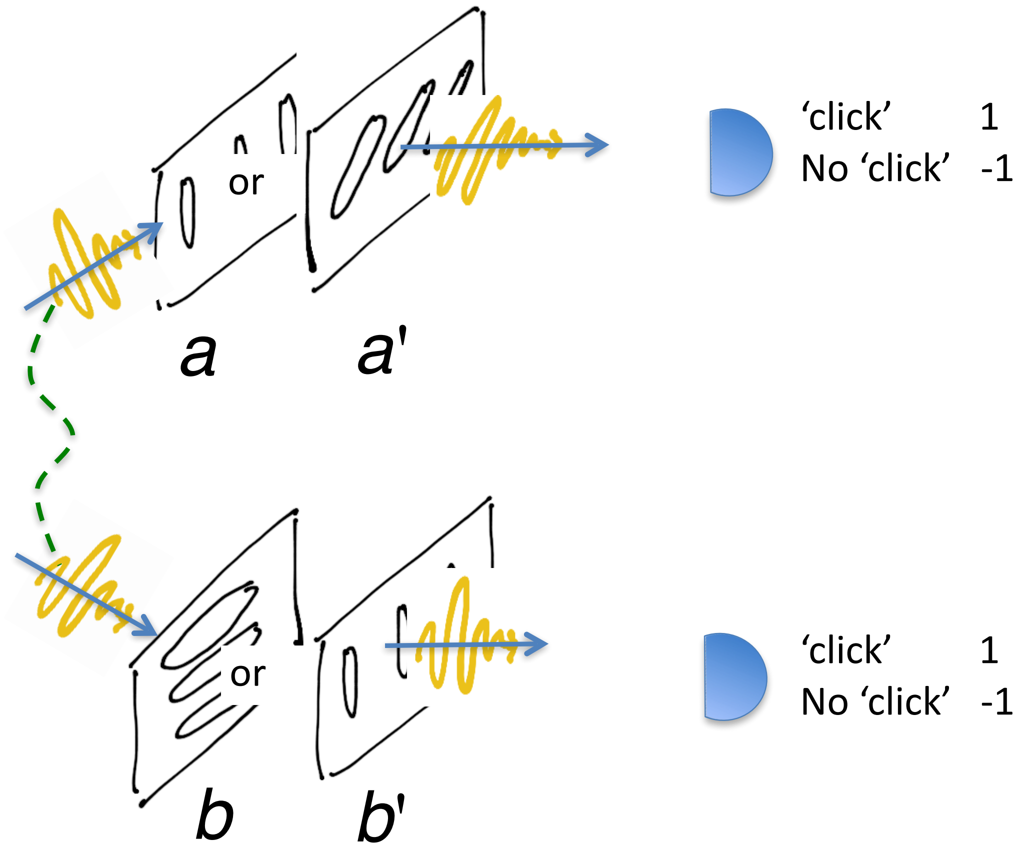


Use entangled pair to test!

Single, but distant, measurements

Locality => canNOT change state

Bell's theorem



Bell: ANY Local Hidden Variable model

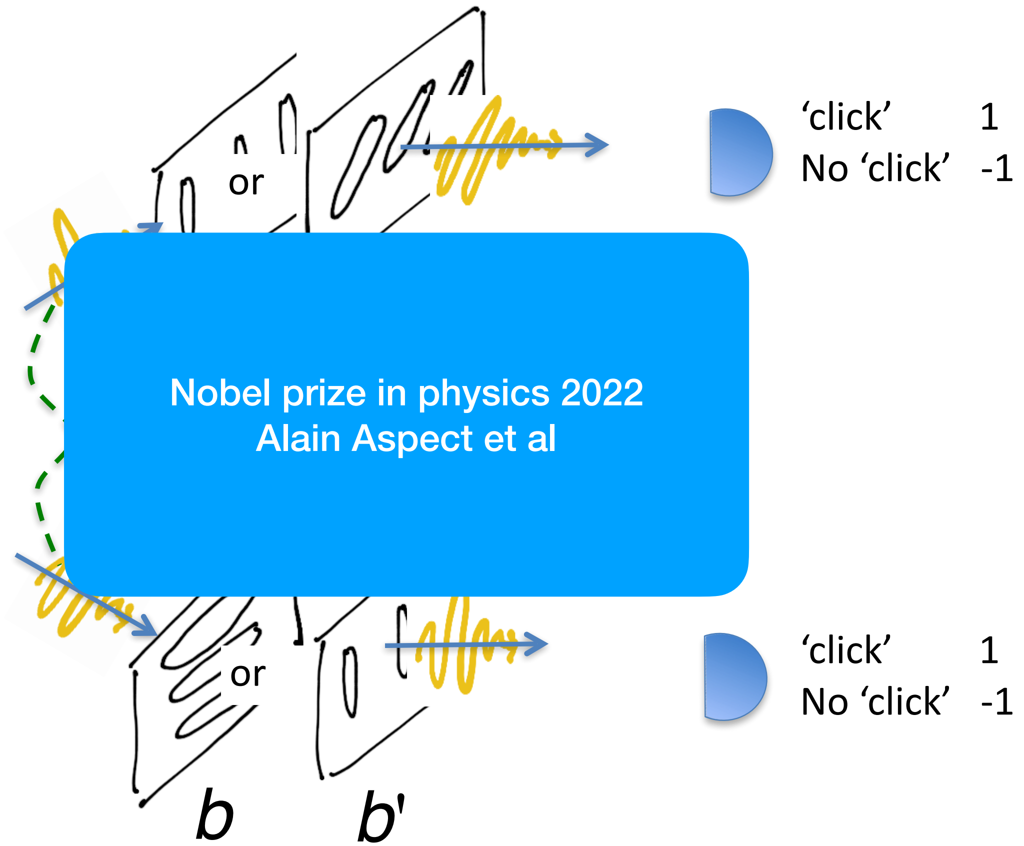
(i.e. where *some* theory knows the outcome)

$$S = \left| a.b + a.b' + a'.b - a'.b' \right| \leq 2$$

QM gives!

=2√2

Bell's theorem



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There is no way to assign ‘value’ to the polarisation and get a deterministic outcome

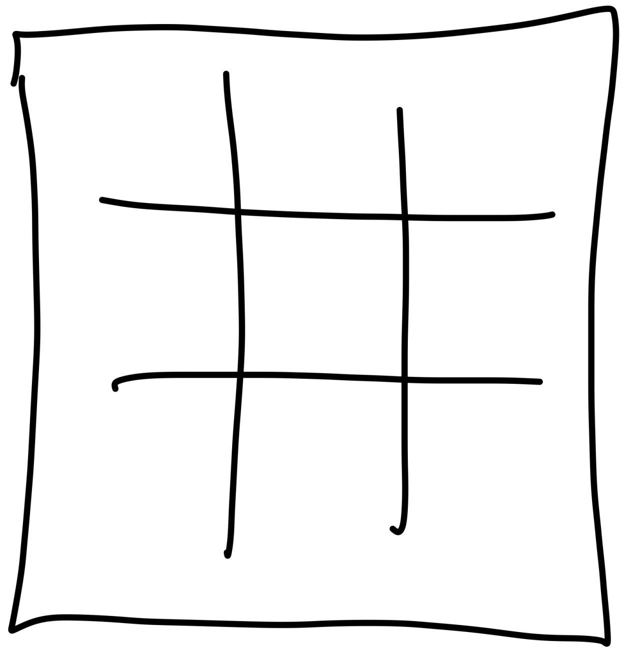
Peres-Mermin magic square game

Quantum randomness is different from classical 'randomness'

↓

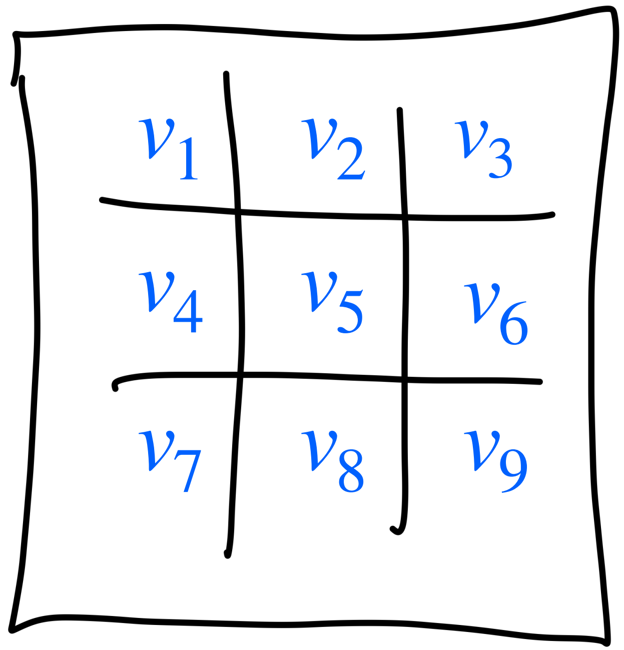
Games that 'classical' devices cannot win, but quantum can

Peres-Mermin magic square game



Peres-Mermin magic square game

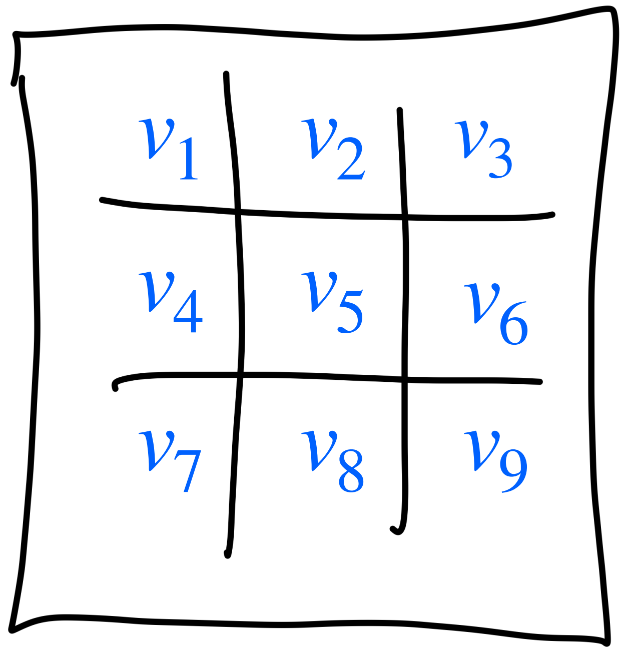
- Player assigns values to all squares in grid
  $v_i = \pm 1$



Peres-Mermin magic square game

- Player assigns values to all squares in grid
  $$v_i = \pm 1$$

- Referee chooses a column or a row, at random, and reads the the product of the values



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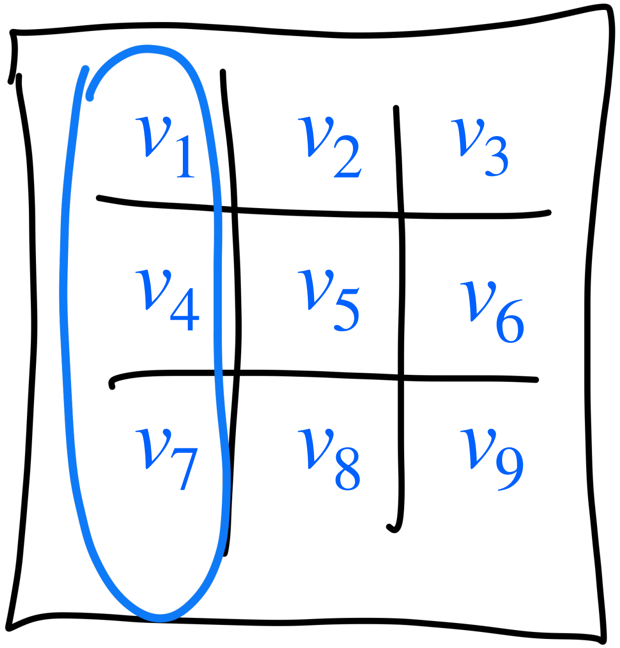

$$c_1 = v_1 \,.\, v_4 \,.\, v_7$$

Peres-Mermin magic square game

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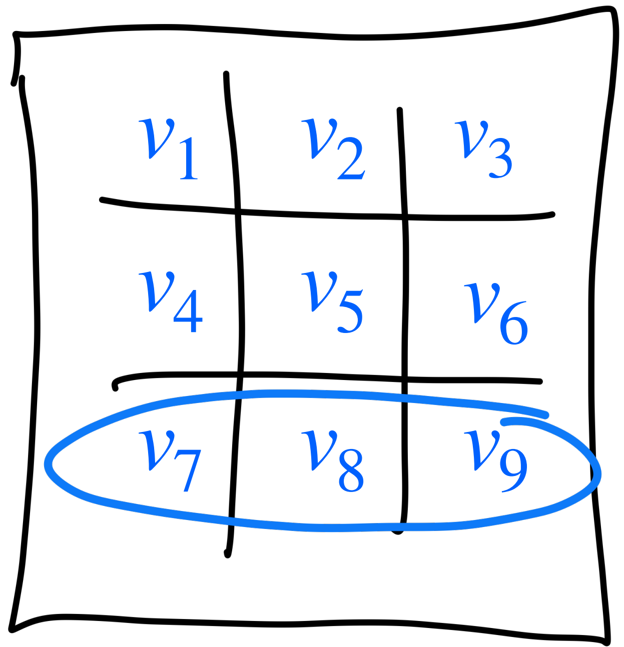
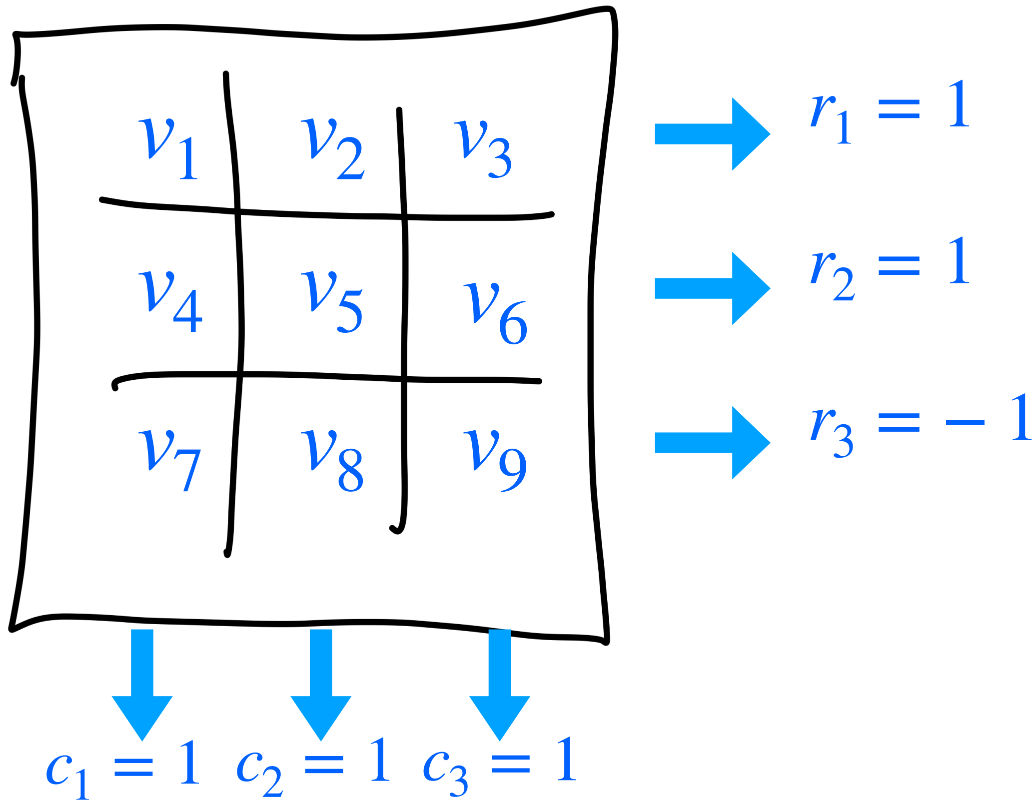


$r_3 = v_7 \, . \, v_8 \, . \, v_9$

Peres-Mermin magic square game

- Player assigns values to all squares in grid $v_i = \pm 1$
- Referee chooses a column or a row, at random, and reads the the product of the values
- Player wins if

$$c_1 = v_1 . v_4 . v_7 = 1$$
$$c_2 = v_2 . v_5 . v_8 = 1$$
$$c_3 = v_3 . v_6 . v_9 = 1$$
$$r_1 = v_1 . v_2 . v_3 = 1$$
$$r_2 = v_4 . v_5 . v_6 = 1$$
$$r_3 = v_7 . v_8 . v_9 = -1$$



Peres-Mermin magic square game

- Player assigns values to all squares in grid
  $v_i = \pm 1$

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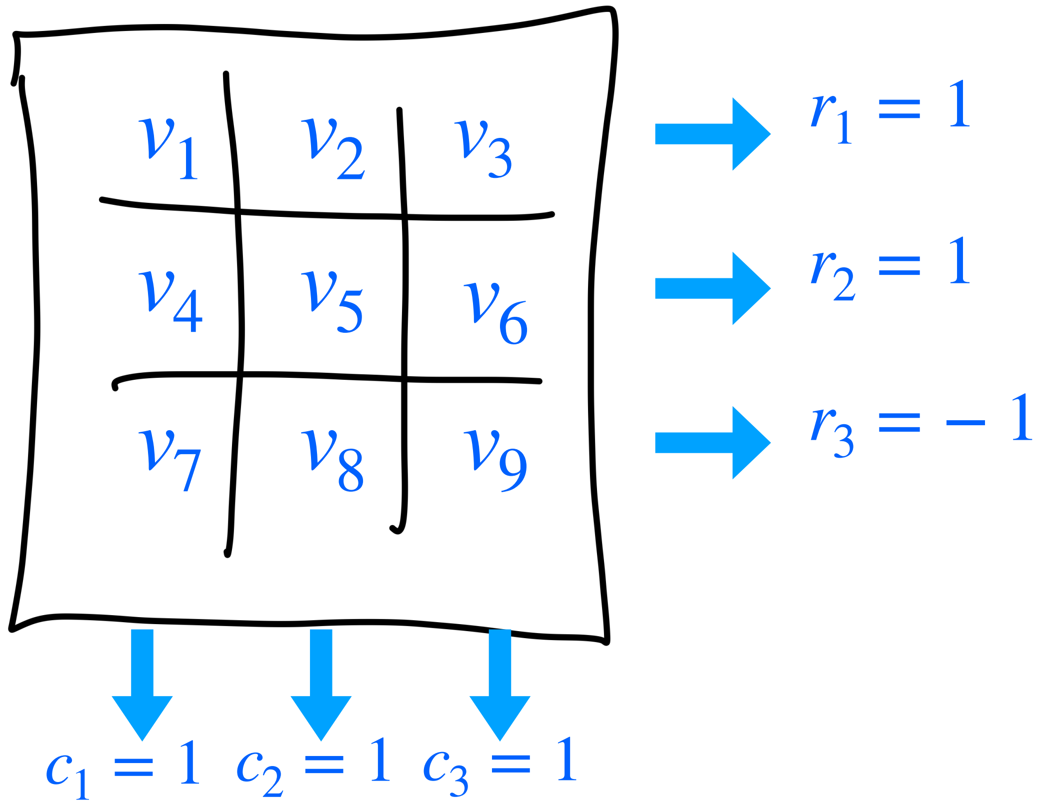
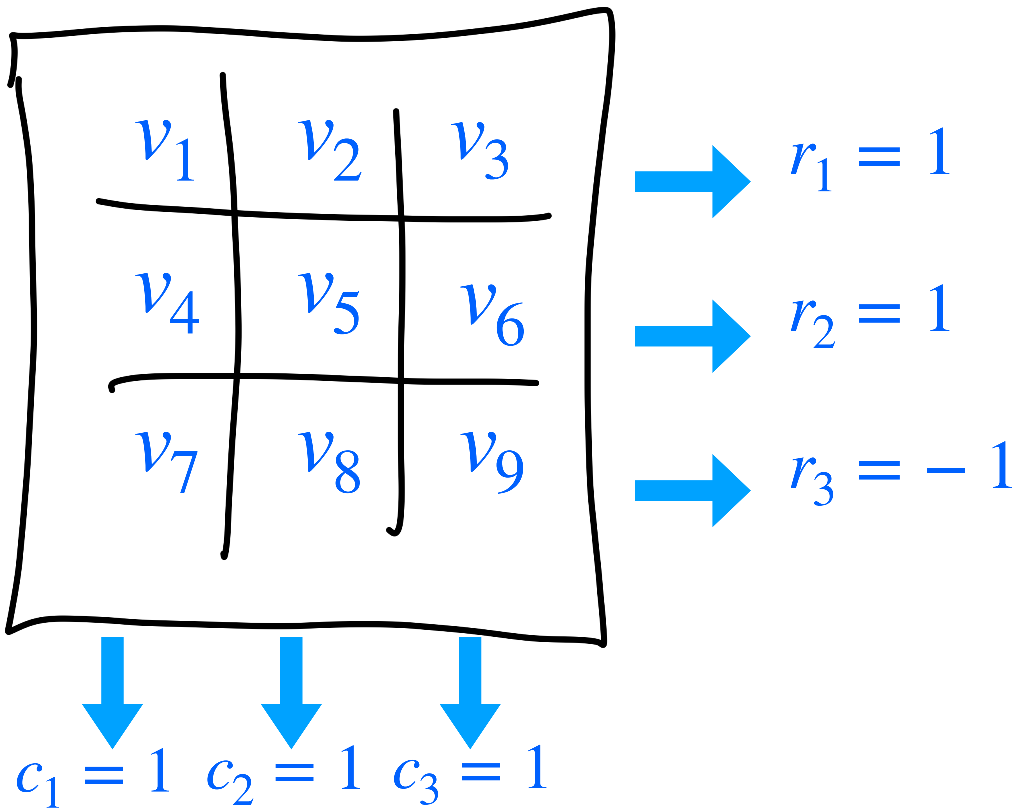
- Player wins if

$$\begin{aligned} c_1 &= v_1 \,.\, v_4 \,.\, v_7 = 1 \\ c_2 &= v_2 \,.\, v_5 \,.\, v_8 = 1 \\ c_3 &= v_3 \,.\, v_6 \,.\, v_9 = 1 \\ r_1 &= v_1 \,.\, v_2 \,.\, v_3 = 1 \\ r_2 &= v_4 \,.\, v_5 \,.\, v_6 = 1 \\ r_3 &= v_7 \,.\, v_8 \,.\, v_9 = -1 \end{aligned}$$



$$p(win) = \frac{1}{6}\left(p(c_1 = 1) + p(c_2 = 1) + p(c_3 = 1) + p(r_1 = 1) + p(r_2 = 1) + p(r_3 = -1)\right)$$

Best possible $p(win)$?



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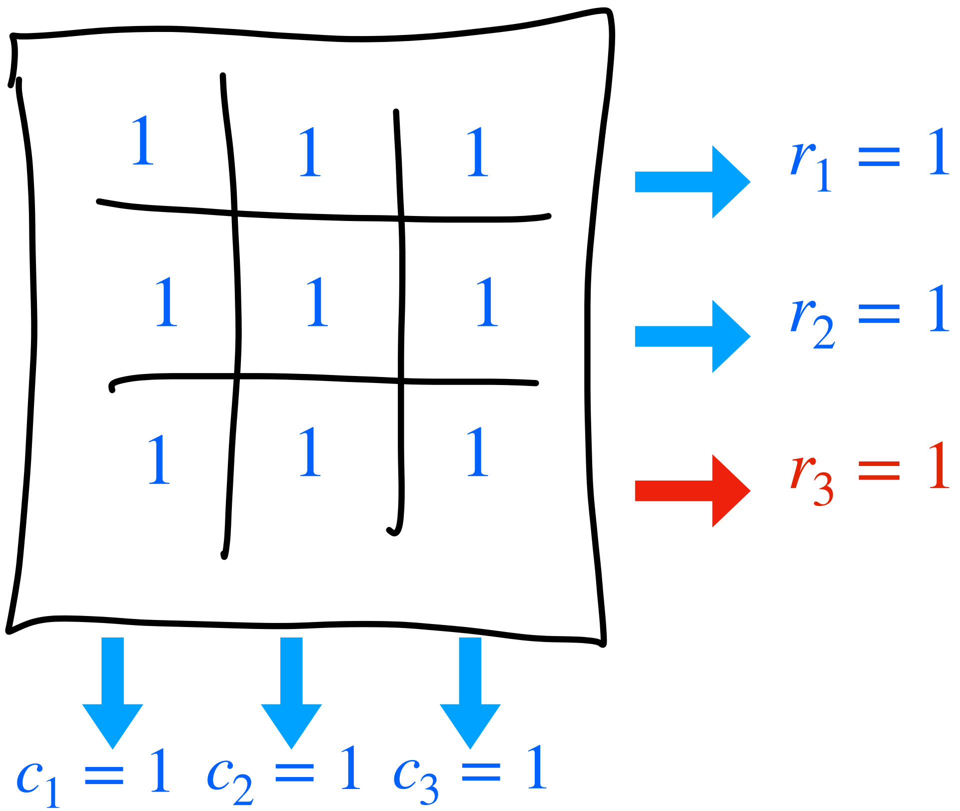

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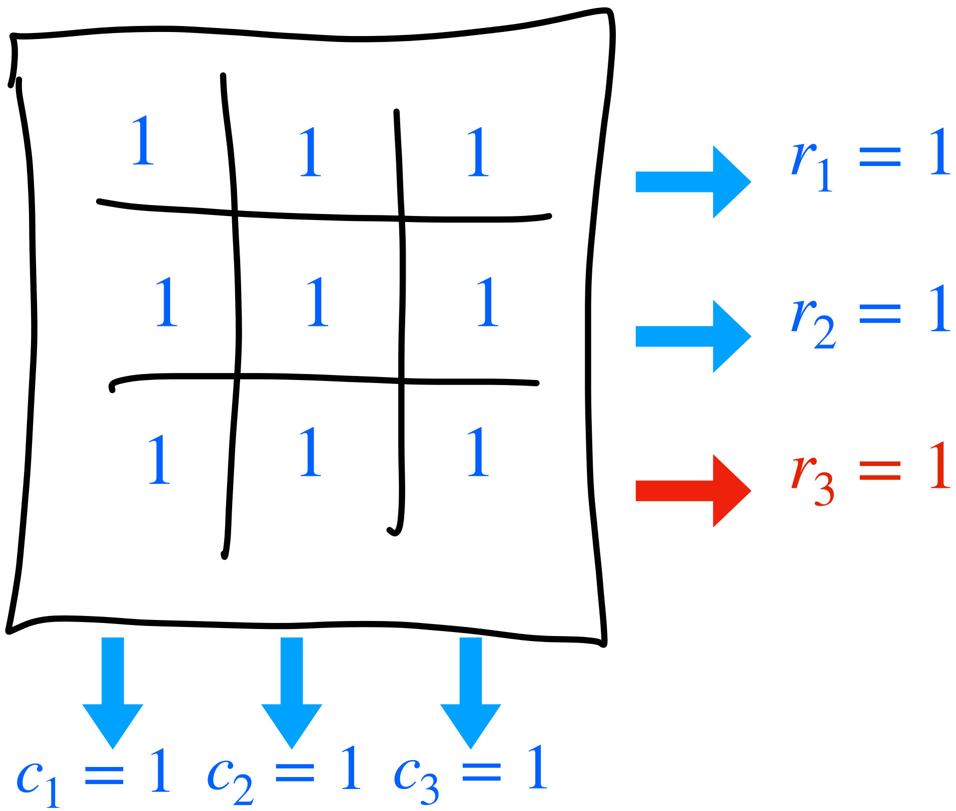
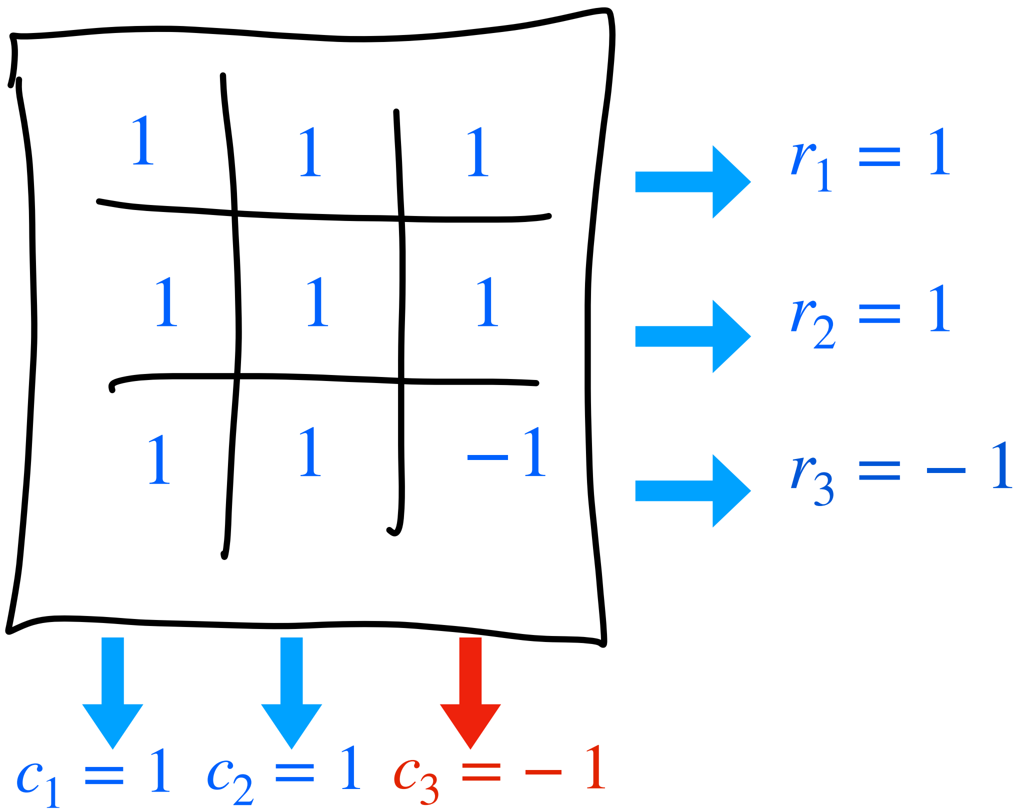


$$p(win) = \frac{5}{6}$$

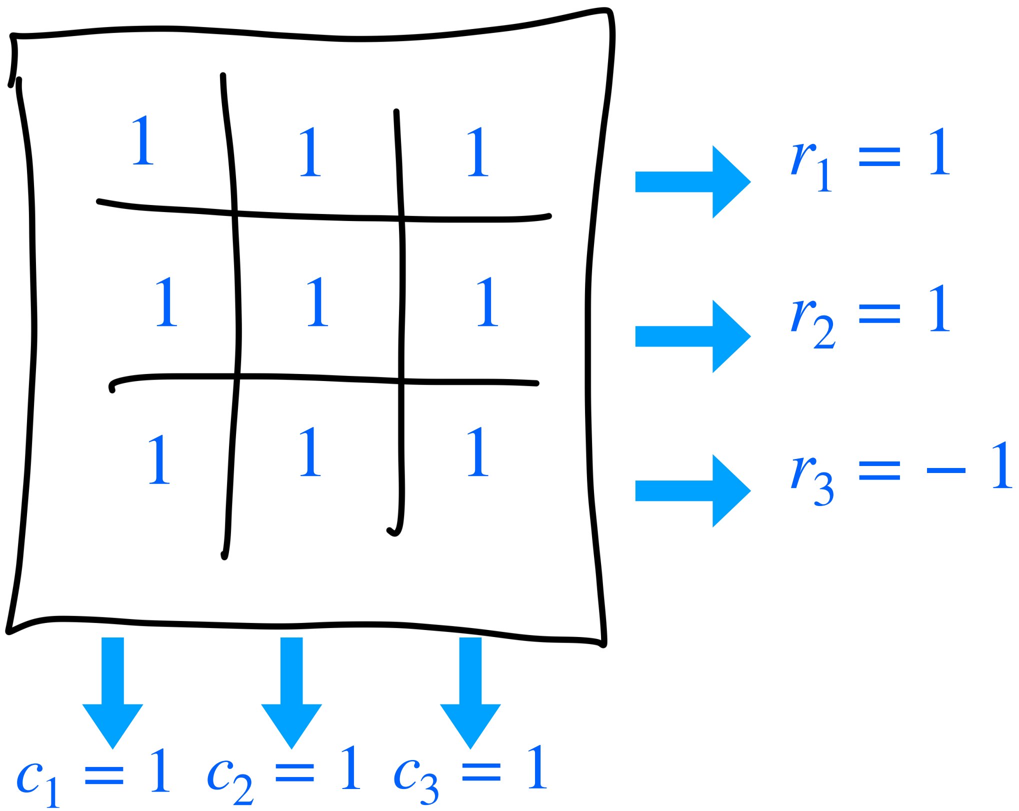
Best possible $p(win)$?



$$p(win) = \frac{5}{6}$$

Best possible $p(win)$?

- Any fixed (deterministic) assignment can only satisfy 5/6 winning conditions



Best possible $p(win)$?

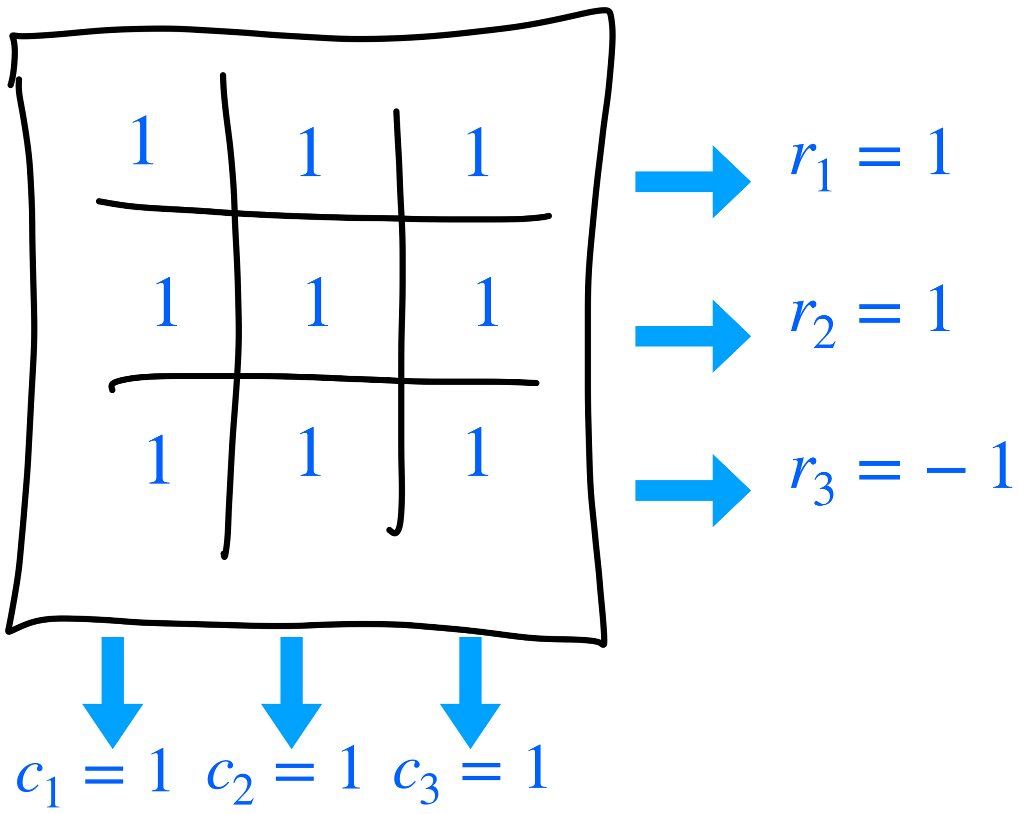
- Any fixed (deterministic) assignment can only satisfy 5/6 winning conditions

$$c_1 . c_2 . c_3 = r_1 . r_2 . r_3$$

Incompatible with

$$c_1 = c_2 = c_3 = r_1 = r_2 = 1$$
$$r_3 = -1$$

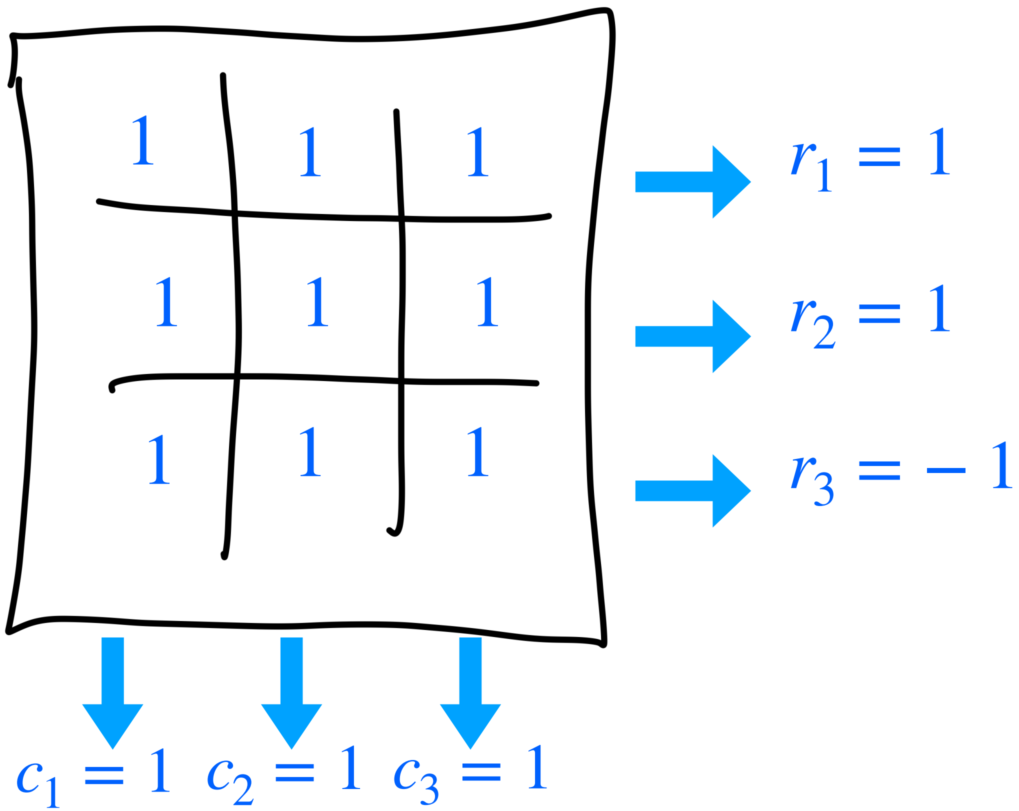
Cannot always win!



Best possible $p(win)$?

- Any fixed (deterministic) assignment can only satisfy 5/6 winning conditions

- Any randomized assignment can only do as well as the best deterministic assignment



Best possible classical $p(win)$?

- Any fixed (deterministic) assignment can only satisfy 5/6 winning conditions

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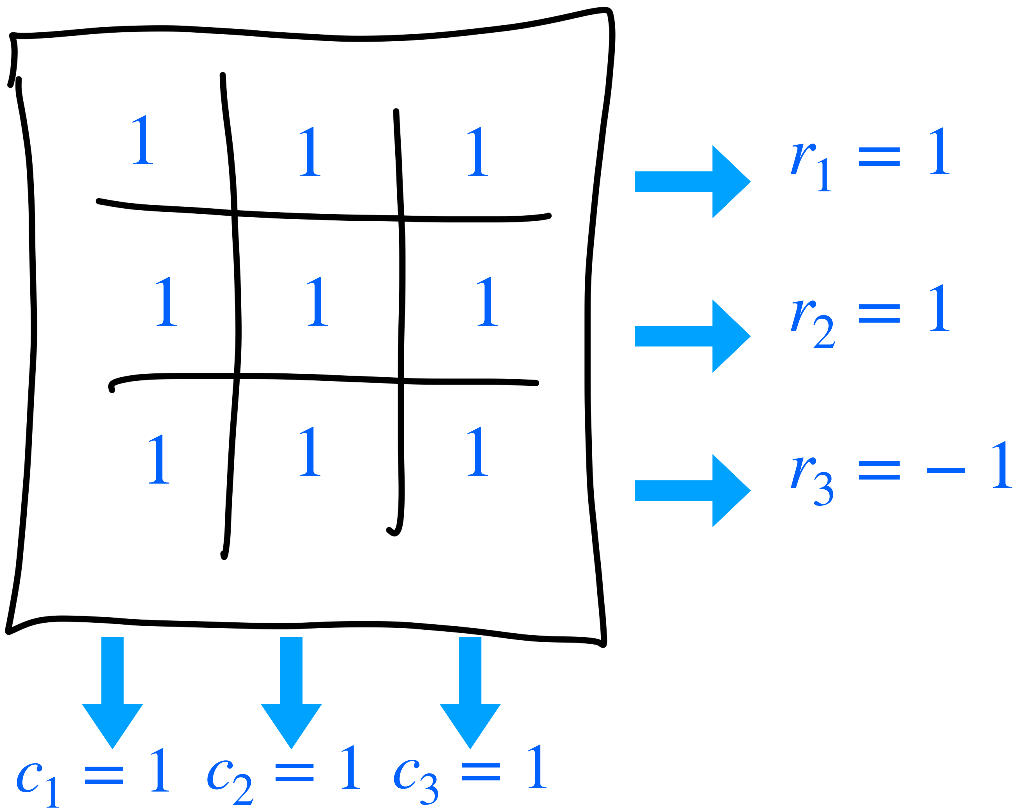
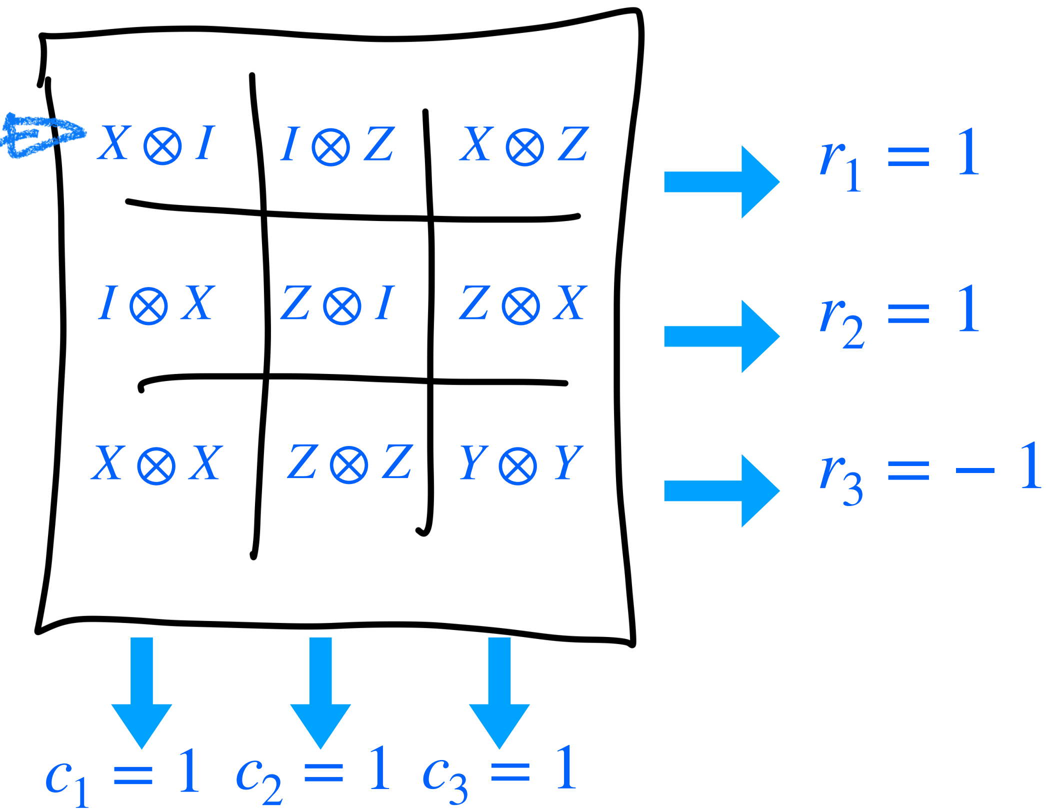


$$p_c(win) \leq \frac{5}{6}$$

Best quantum $p(win)$?

- Quantum measurements with outcomes = ± 1
- Can be co-measured in any row or column
- Satisfy all winning conditions!



$$p_Q(win) = 1$$

Best quantum $p(win)$?

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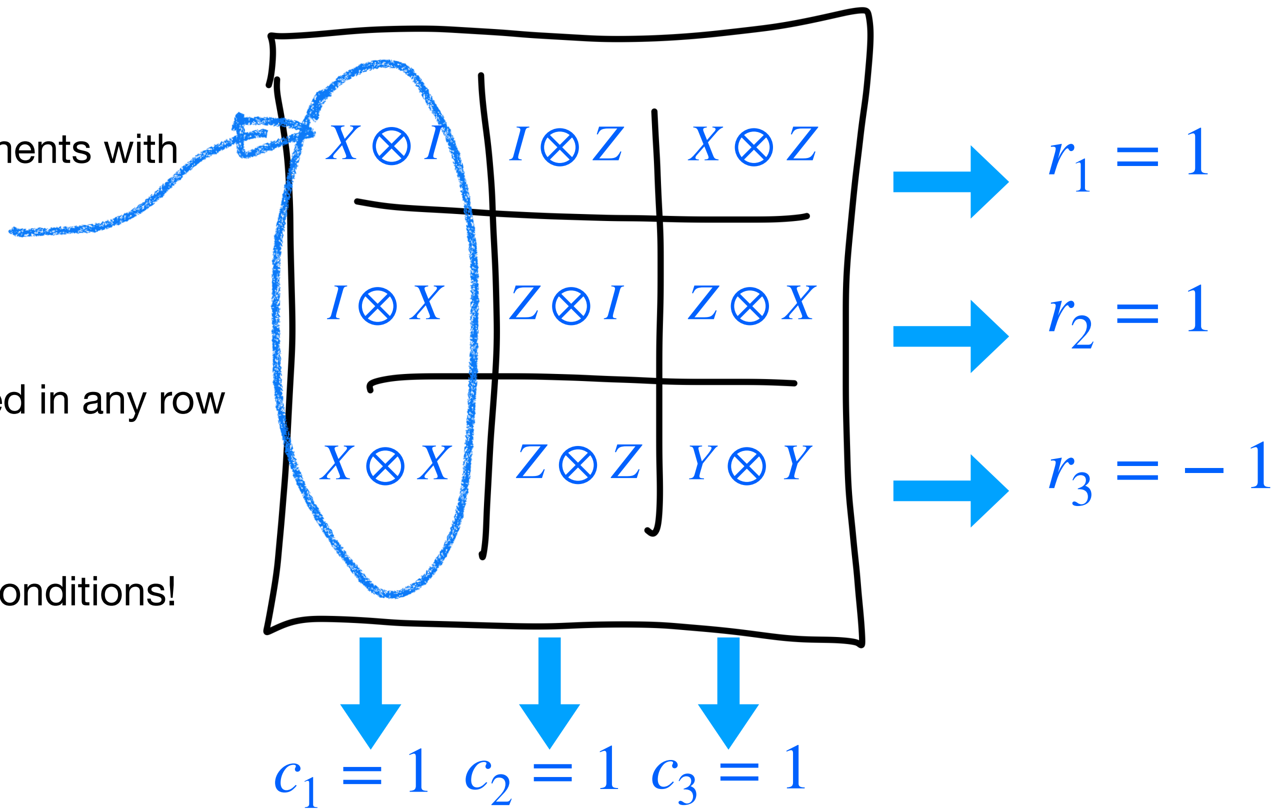

$p_Q(win) = 1$

Best quantum $p(win)$?

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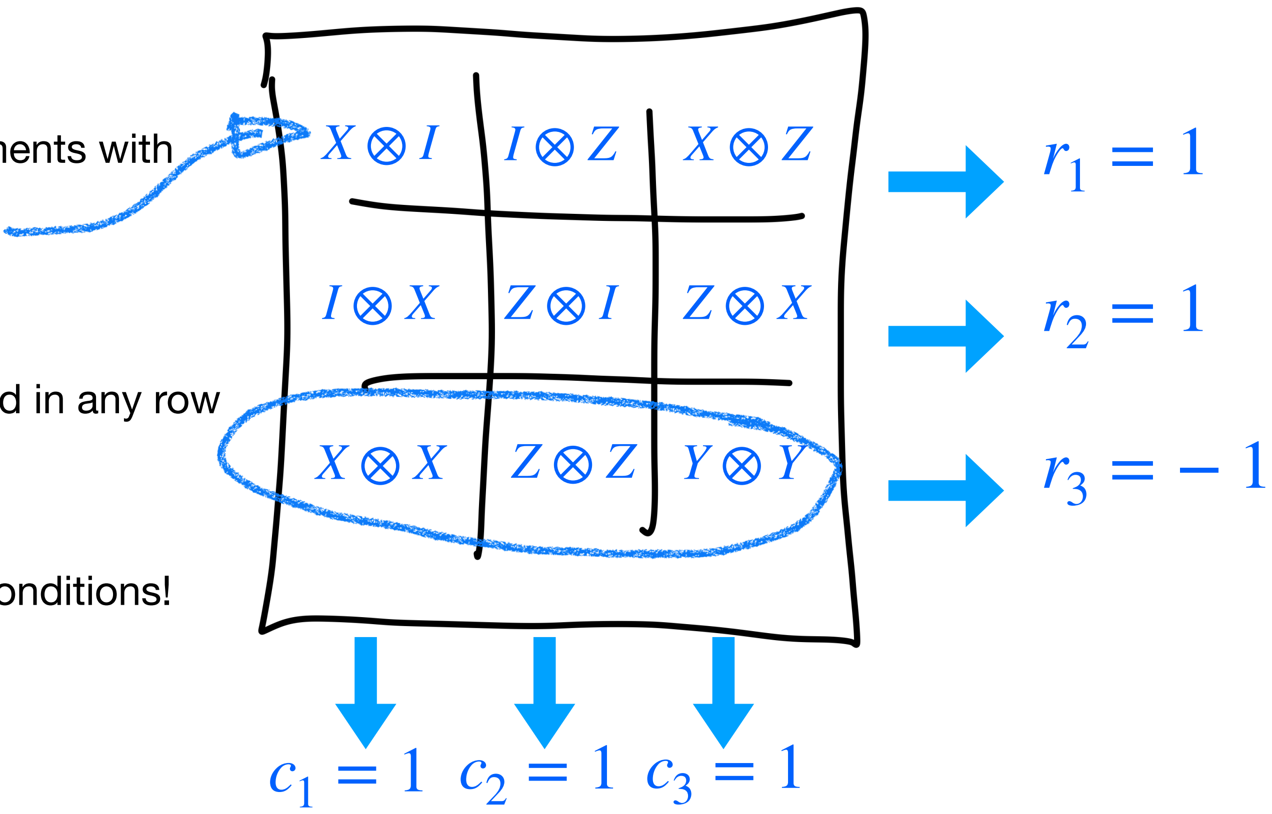
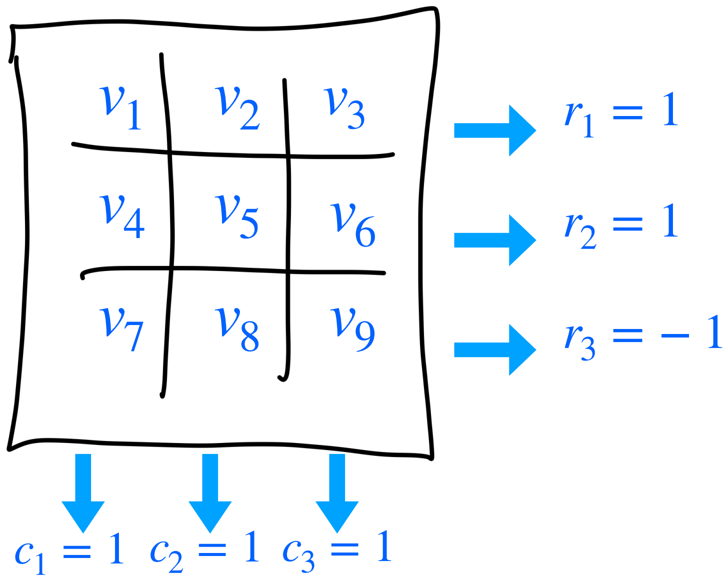


$p_Q(win) = 1$

Peres-Mermin magic square game



$$p_c(win) \leq \frac{5}{6}$$

$$p_Q(win) = 1$$

- Constraints not achievable classically, can achieve quantumly
- Directe applications to shallow circuit, provable quantum advantage [Bravyi, Gosset, Koening, Science 2017]

Behind all quantum computational advantage?

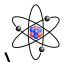
2. What is quantum computing?

2. What is quantum computing?

Bit

0 / 1

Qubit



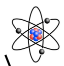
$$\alpha\left|0\right\rangle + \beta\left|1\right\rangle$$

2. What is quantum computing?

Bit

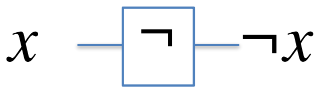
0 / 1

Qubit

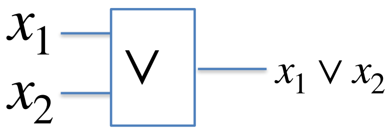


$$\alpha|0\rangle + \beta|1\rangle$$

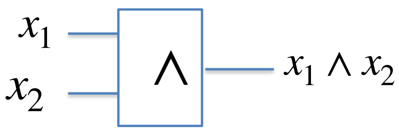
Gates



Not

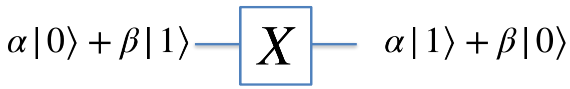


OR



AND

Unitary gates



Not

Unitary map

(Reversible)

2. What is quantum computing?

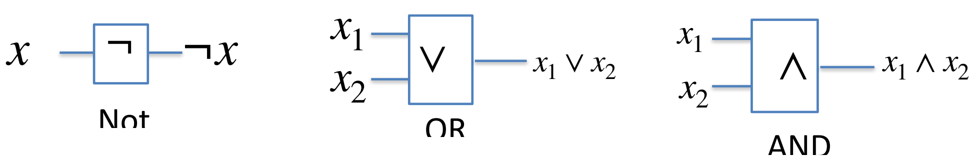
Bit

0 / 1

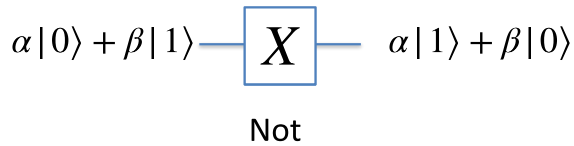
Qubit

$$\alpha|0\rangle + \beta|1\rangle$$

Gates

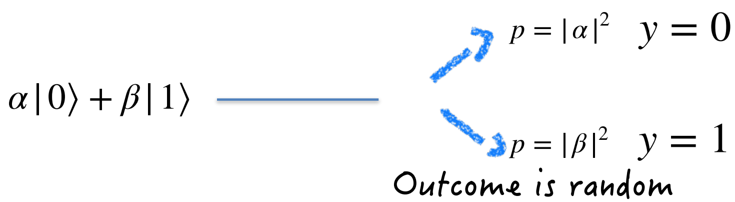


Unitary gates



Read out

Readout measurements



2. What is quantum computing?

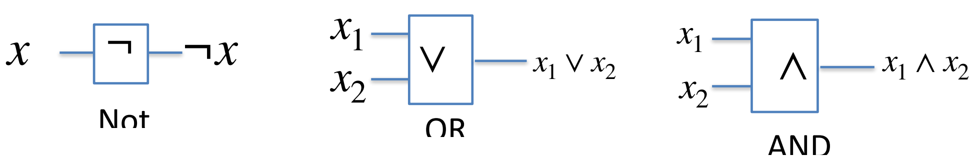
Bit

0 / 1

Qubit

$\alpha|0\rangle + \beta|$

Gates



Unitar

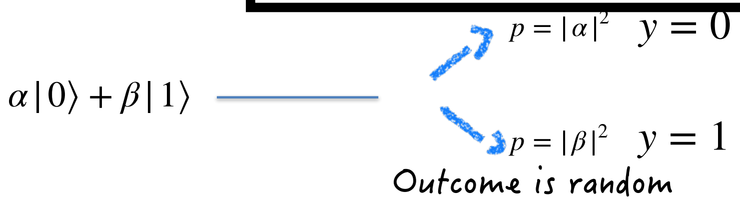
$\alpha|0\rangle + \beta|1\rangle$

Note:
Just linear algebra!

Read out

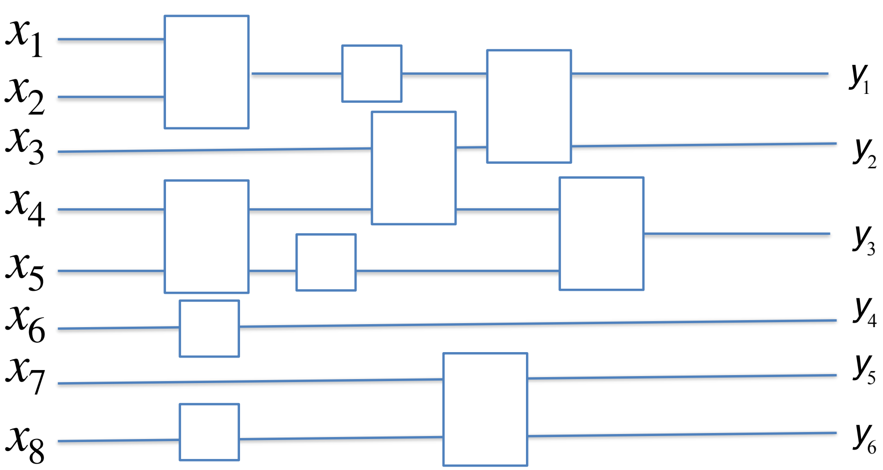
x

Reado



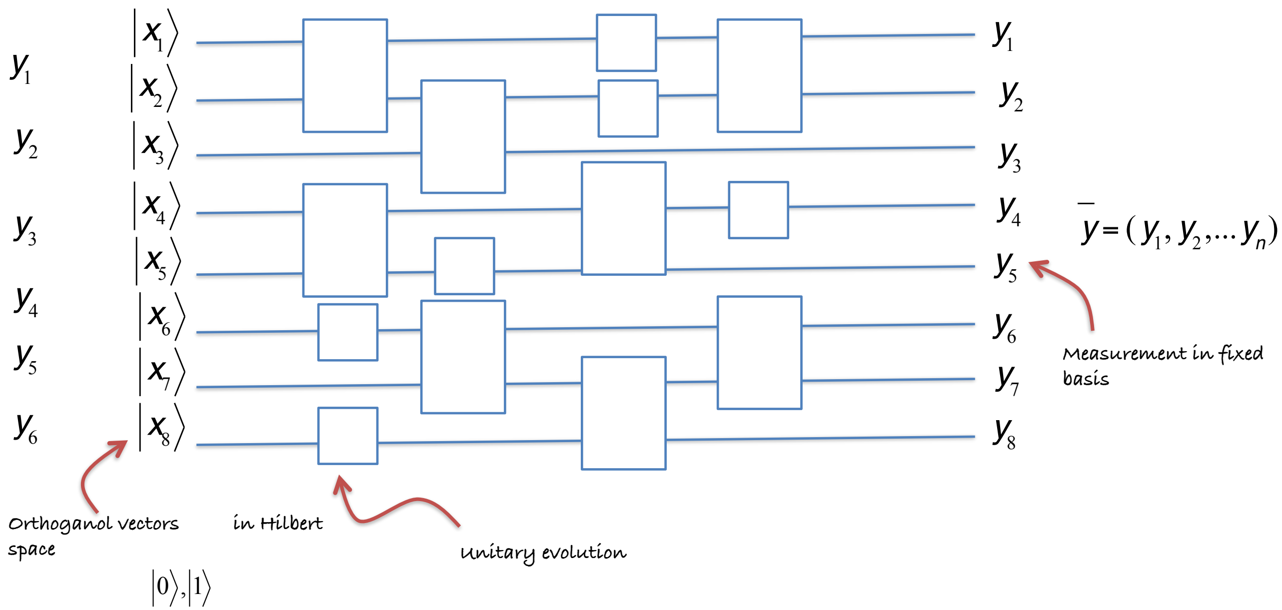
2. What is quantum computing?

Circuit model



Universal gate set: NOT, AND, OR

Quantum circuit model



Universal quantum gate set: CNOT, pi/8, H

Complexity classes for quantum computing

Decision problems:
Functions from bit strings length n to single bit

$$f : \{0,1\}^n \rightarrow \{0,1\}$$

Language L:
Set of inputs which output 1

$$x \in L \quad \text{iff} \quad f(x) = 1$$

BPP

$L \in$ BPP if $\exists$ a family of circuits $\{C_n\}$ such and a polynomial $q(n)$ such that

- size of circuits $|C_n| \leq q(n)$
- If $x \in L$, output 1 with probability > 2/3
- If $x \notin L$, output 1 with probability <1/3

BQP

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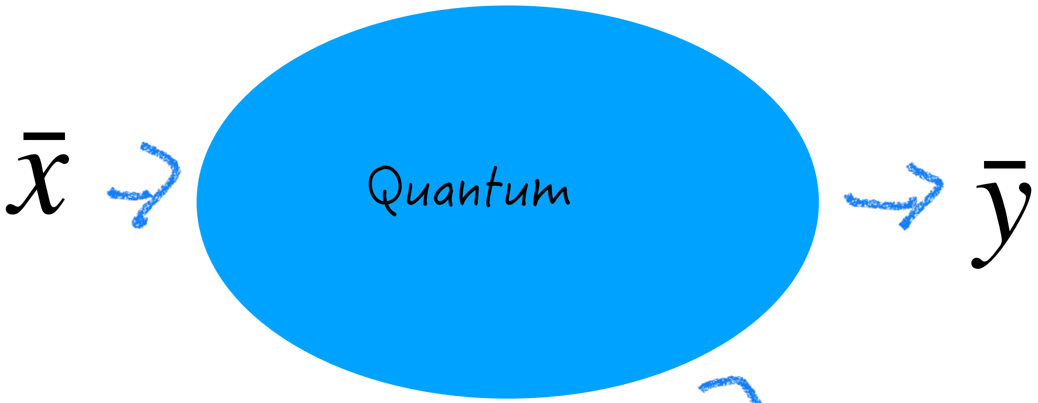
Other models of quantum computation?



Measurement based quantum computation

Adiabatic quantum computation

…

Complexity-wise equivalent to circuit model

Big conjecture of quantum computing

BPP $\subset$ **BQP**

Not proven…

3. What can we do with quantum computers?

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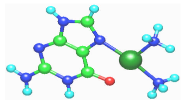
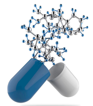
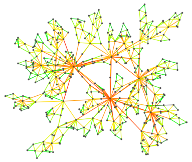
Factoring primes (cracks RSA)

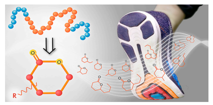
Quantum Machine Learning

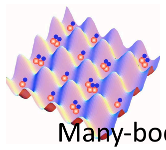

Quantum Chemisty


Drug development


Search


Quantum random walks


Material design


Many-body physics

10 or 40-70 qubits

-> 100 qubits beyond classical

3. What can we do with quantum computers?

Exponential 'improvement'

- Shor's factoring algorithm
  [Shor, FOCS 1994]
  Factors numbers into primes
  -> could 'crack' RSA

- System of linear equations
  [Harrow, Hassidim, Lloyd, PRL 2008]
  Notable application to machine learning
  -> needs 'QRAM'
  -> applications?

'Proven' quantum advantage

- Sampling problems
  [Aaronson, Arkhipov 2013]
  [Bremner, Josza, Shepherd 2011]
  Boson sampling, IQP, random shallow circuits…
  -> efficient classical simulation
  => collapse of PH
  -> applications?

- Shallow circuit advantage
  [Bravyi, Gosset, König, Science, 2018]
  Constant depth Q requires log depth C
  -> PROOF: concenquence of 'Q randomness'
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Variational, ML, ...

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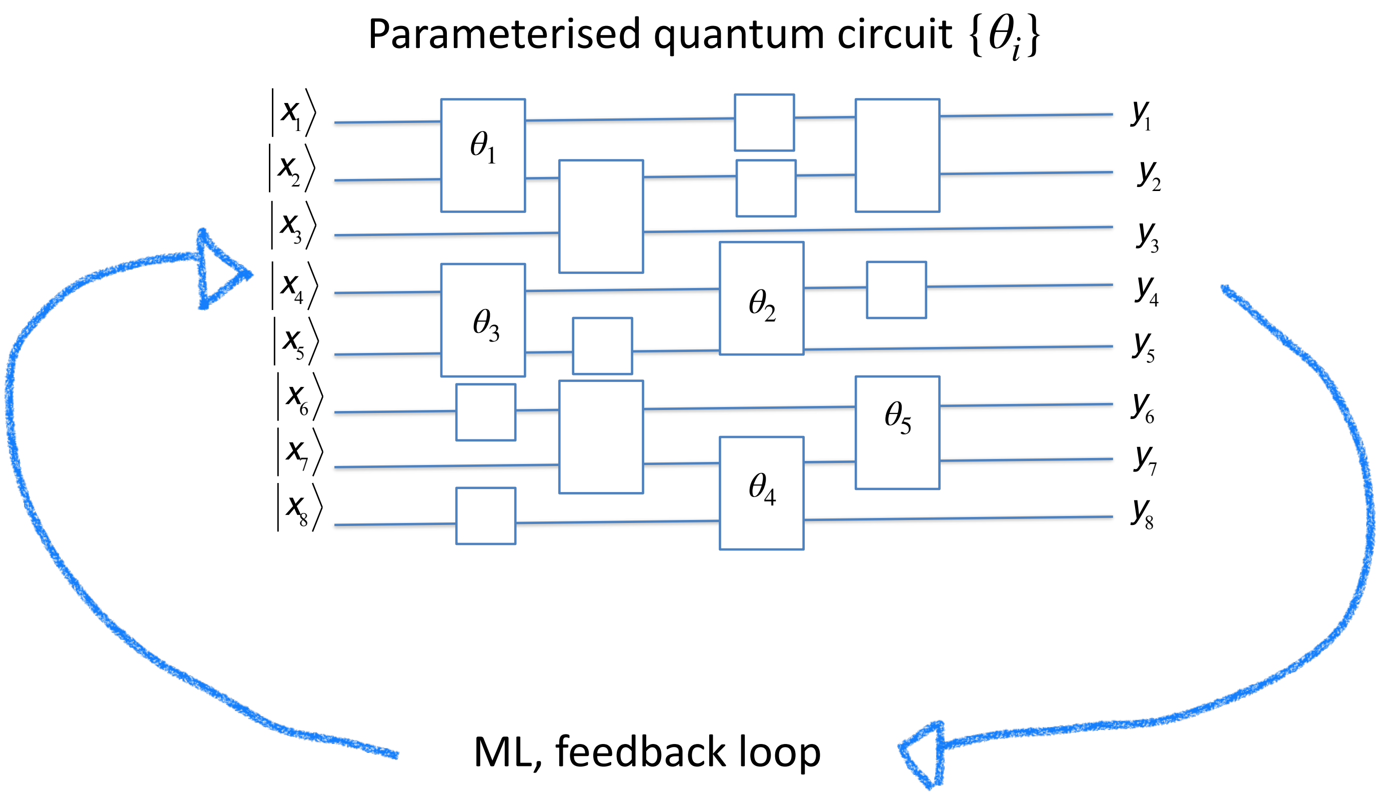
Variational quantum circuits, ML, ...



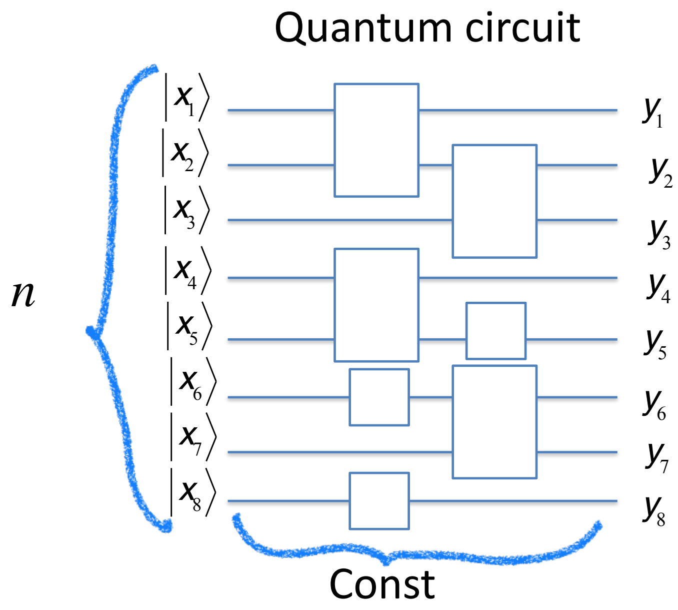
-> Quantum chemistry

-> Machine learning

-> ...

Shallow quantum circuits

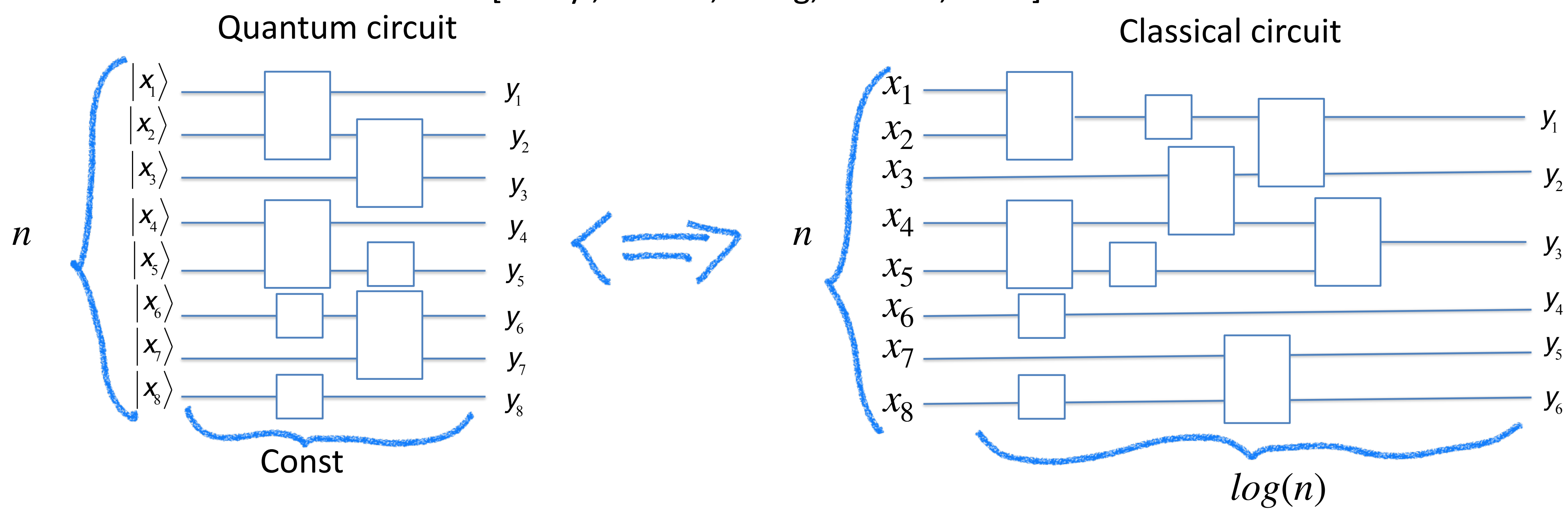
[Bravyi, Gosset, König, Science, 2018]



Relational statement $R(\bar{x}, \bar{y})$

Shallow quantum circuits

[Bravyi, Gosset, König, Science, 2018]



Relational statement $R(\bar{x}, \bar{y})$

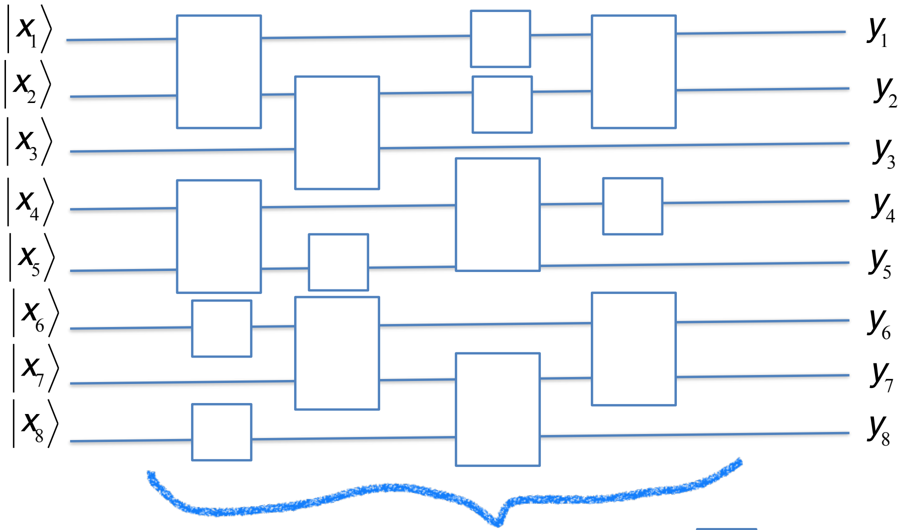
Impossible to satisfy classically in constant circuit

-> 'Circuit' magic square game…

Sampling

[Bremner, Josza, Shepherd 2011]

Subuniversal circuit families



All gates commute with X

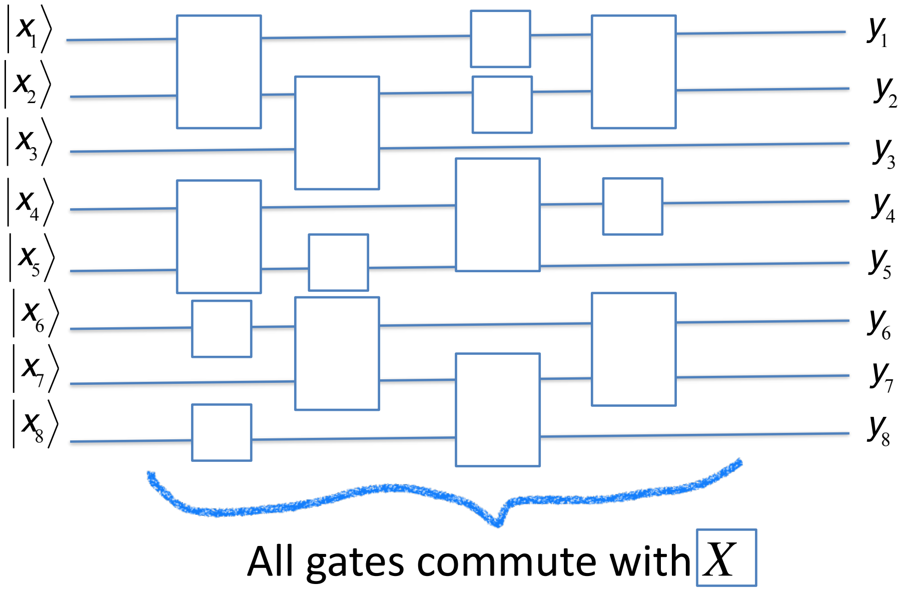
Probabilistic output:
$\bar{y}$ with prob $p(\bar{y})$

No classical poly circuit outputting $\bar{y}$ $p(\bar{y})$ else PH collapses

Sampling

[Bremner, Josza, Shepherd 2011]

Subuniversal circuit families



Probabilistic output:
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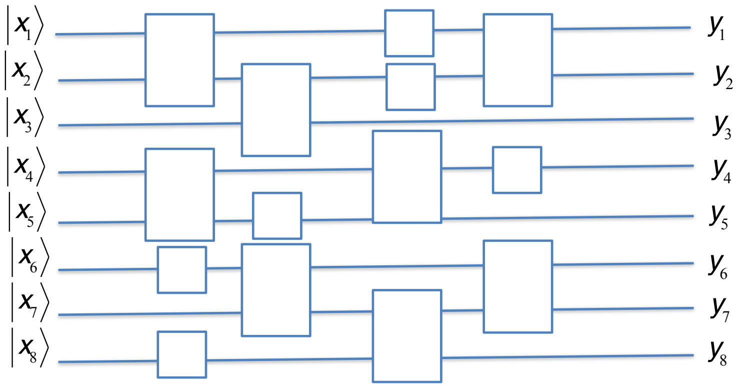
Quantum randomness at play here? Links to shallow circuit?

Sampling hardness implies shallow circuit

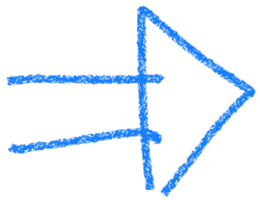
Sampling

Subuniversal circuit families

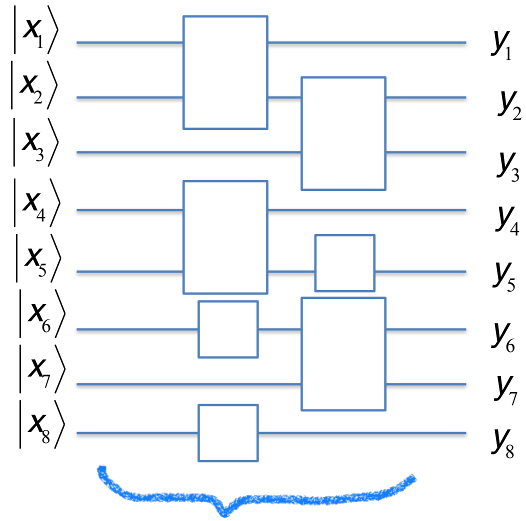


$\bar{y}$

$p(\bar{y})$



Shallow circuit advantage



Const

No classical poly circuit outputting $\bar{y}$ $p(\bar{y})$
else PH collapses

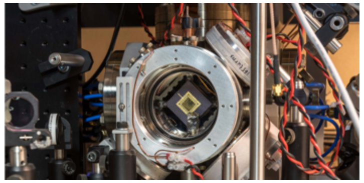
Relational statement $R(\bar{x}, \bar{y})$

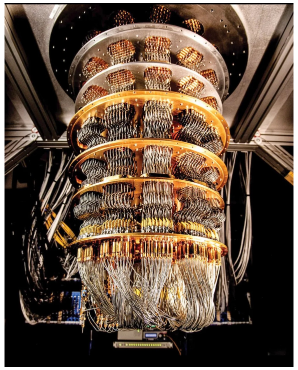
Impossible to satisfy classically in constant circuit

4. What's so hard about building a quantum computer?

Quantum coherence is fragile

- Decoherence, limits to 'classical'
- Require huge control and optimisation …





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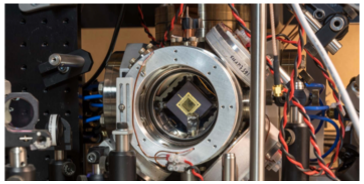

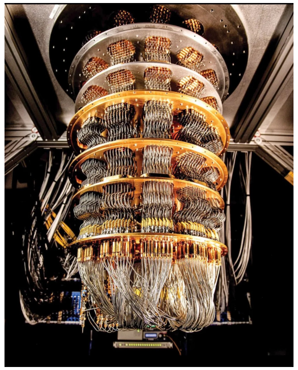

Quantum error correction and Fault Tolerance

- Possible!: more systems, more steps, feedback
- Huge overhead…

Quantum
the open journal for quantum science

PAPERS PERSPECTIVES

How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney[1] and Martin Ekerå[2,3]

[1]Google Inc., Santa Barbara, California 93117, USA
[2]KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden
[3]Swedish NCSA, Swedish Armed Forces, SE-107 85 Stockholm, Sweden

20^6 noisy qubits to factor 2048 bits

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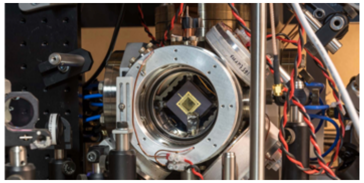

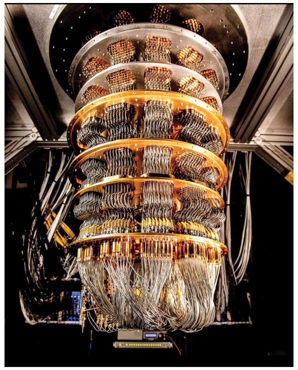

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20^6 noisy qubits to factor 2048 bits

Challenge: good codes/schemes, that work with real systems…

What can we do with quantum computers?

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Require FTQC

?

Variational, ML, …

'NISQ'
How much noise
FT?

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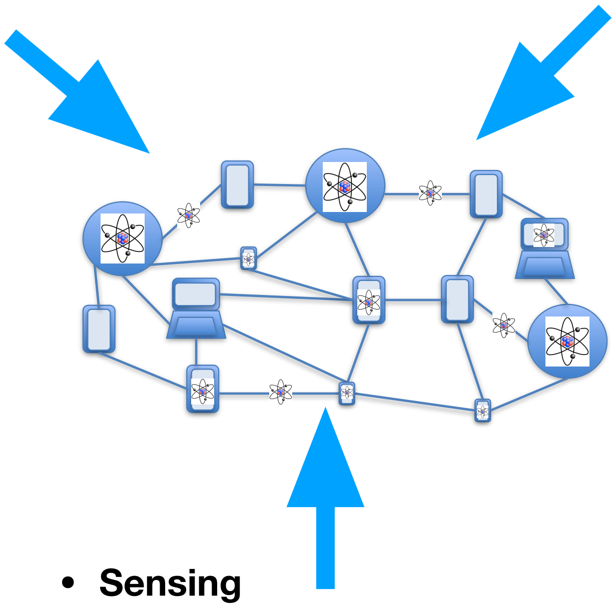
6. Quantum Networks

- **Computation**
  Exponential speed up (Shor), QML

- **Communication**
  Security (QKD), communication complexity



- **Sensing**
  Sensitivity and precision measurements impossible classically

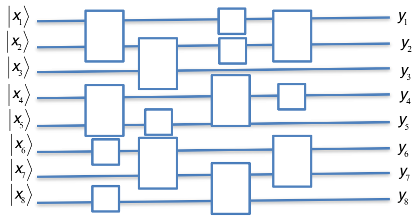
FRANCE 2030

QUANTUM FLAGSHIP

QIA QUANTUM INTERNET ALLIANCE

Conclusions

Quantum computing is not that complicated…



Just a special linear algebra processor

It's not just Shor's algorithm

Sampling

Shallow circuit

Search

…

Variational, ML, …

It's not just 'quantum computers'